TEERTHANKER MAHAVEER UNIVERSITY
MORADABAD, INDIA

CENTRE FOR ONLINE & DISTANCE LEARNING

Programme: Bachelor of Commerce

Course: Business Mathematics

Course Code: BCPSE201

Semester-II

Syllabus

OBJECTIVE AND EXPECTED OUTCOME OF THE COURSE:

The course consists of instruction in the fundamentals of mathematics as applied to business situations. The course includes the study of fundamental mathematics and calculations which are commonly used in finance and accounting.

UNIT-I

Progressions: Application of Arithmetic Progression and Geometric Progression. Arithmetic progressions finding the 'n'th term of an AP and also sum to 'n' terms of an AP. Insertion of Arithmetic means in given terms of AP and representation of AP. Geometric progression: finding nth term of GP.

UNIT-II

Interest Applications: Simple interest, compound interest including half yearly and quarterly calculation, Instalment Purchases (Cost of Instalment, Effective rates, amortization of a loan)

UNIT-III

Percentage and Ratios' Applications: Percents, Commissions, Discounts, e.g., bill discounting, mark up and concepts of Ratios.

UNIT-IV

Corporate and Special Applications: Computation of the costs and proceeds of stock buyand-sell; Computation of rates of yield and gains or losses on the purchase and sale of stocks; Computation of gains and losses on convertible and callable bonds, annual interest, accrued interest, and annual yield and computation of a rate of yield to maturity

UNIT-V

Foreign Exchange: Brief Introduction to Foreign Exchange Market, Rate of Exchange, Direct/Cross rate and Indirect rate of Exchange, Cross rate, Simple and Compound Conversion, Chain Rule (in the course of exchange)

SUGGESTED READINGS/BOOKS:

- **Business Mathematics and Statistics (Quantitative Techniques for Business):** T R Jain, S C Aggarwal, N Ranade and S K Khurana, (V K *(India)* Enterprises, New Delhi)
- **A textbook of Business Mathematics:** Dr. A. K Arte & R.V. Prabhakar
- **Business Mathematics:** Sanchethi and Kapoor
- **Mathematics for Cost Accountants:** Saha

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UNIT 1 ARITHMETIC PROGRESSION AND GEOMETRIC PROGRESSION

Unit structure

- Introduction
- Arithmetic Progression (A.P.)
- Geometric Progression (G.P.)
- Summary
- Keywords
- Review Questions
- Further Readings

Learning Objectives

After reading this unit you should be able to:

- Define and use arithmetic progression
- Insertion of arithmetic progression
- Define and use geometric progression
- Relation of Mathematics with other disciplines

1.1 INTRODUCTION

A sequence is an arrangement of numbers in a definite order according to some rule. There are several ways of representing a real sequence. One way to represent a real sequence is to list its first few terms till the rule for writing down other terms becomes clear. For example: 1, 3, 5… is a sequence whose *n*th term is (2n -1). Another way is to give a rule of writing the *n*th term of the sequence. For example, the sequence 1, 3, 5, 7 …can be written as a_n = 2n -1. It is not necessary that the terms of a sequence always follow a certain pattern or they are described by some explicit formula for the nth term. *Those sequences whose terms follow certain pattern are called progressions.* This unit deals with arithmetic progression and geometric progression in detail.

1.2 ARITHMETIC PROGRESSION (A.P.)

1.2.1 Arithmetic Progression

A sequence is called an arithmetic progression if the difference of a term and the previous term is always same, i.e.

$$a_{n+1} - a_n = \text{constant } (=d) \text{ for all } n \in N$$

The constant difference, generally denoted by d is called the common difference. Thus, if the first term is *a* and the common difference is *d*, then

$a, a+d, a+2d, a+3d, a+4d, \ldots\ldots\ldots\ldots\ldots\ldots$

is an arithmetic progression.

Applications of A.P. in business

Illustration 1.1

The sequence 1, 4, 7, 10, 13,…is an A.P. whose first term is 1 and the common difference is equal to 4 -1 = 3.

In order to determine whether a sequence is an A.P. or not when its *n*th term is given, we may use the following algorithm:

1.2.2 Algorithm

STEP I Obtain a_n
STEP II Replace n by $n + 1$ in a_n to get a_{n+1}
STEP III Calculate $a_{n+1} - a_n$
STEP IV If $a_{n+1} - a_n$ is independent of n, the given sequence is an A.P. Otherwise, it is not an A.P.

Illustration 1.2 *Show that the sequence* $\langle a_n \rangle$ *defined by* $a_n = 4n + 5$ *is an A.P. Also, find the common difference.*

Solution:

We have, $a_n = 4n + 5$

Replacing n by $(n+1)$, we get

$a_{n+1} = 4(n+1) + 5 = 4n + 9$

Now, $a_{n+1} - a_n = (4n+9) - (4n+5) = 4$

Clearly, $a_{n+1} - a_n$ is independent of n and is equal to 4. So, the given sequence is an A.P. with common difference 4.

Illustration 1.3 Show that the sequence $\langle a_n \rangle$ defined by $a_n = 2n^2 + 1$ is not an A.P.

Solution:

We have, $a_n = 2n^2 + 1$

Replacing n by (n + 1) in a_n, we obtain

$$a_{n+1} = 2(n+1)^2 + 1 \Rightarrow a_{n+1} = 2n^2 + 4n + 3$$

Now, $a_{n+1} - a_n = (2n^2 + 4n + 3) - (2n^2 + 1) = 4n + 2$

Clearly, $a_{n+1} - a_n$ is not independent of *n* and is therefore, not constant. So, the given sequence is not an A.P.

Illustration 1.4 Show that the sequence $<a_n>$ is an A.P. if its *nth* term is a linear expression in *n* and in such a case the common difference is equal to the coefficient of *n*.

Solution:

Let $<a_n>$ be a sequence such that its *n*th term is a linear expression in *n* i.e.

$a_n = A_n + B$, where A, B are constants.

$\Rightarrow$ $a_{n+1} = A(n+1) + B$

$\therefore a_{n+1} - a_n = \{A(n+1) + B\} - \{An + B\} = A$

Clearly, $a_{n+1} - a_n$ is independent of *n* and is $\therefore$ a constant. So, the sequence $<a_n>$ is an A.P. with common difference A.

Illustration 1.5 The *n*th term of a sequence is 3n – 2. Is the sequence an A.P.? If so, fins its 10th term.

Solution:

Here, $a_n = 3n - 2$.

Clearly, a_n is a linear expression in *n*. So, the given sequence is an A.P. with common difference 3.

Putting n = 10, we get

$a_{10} = 3 \times 10 - 2 =$ **28**

NOTE: A sequence is not an A.P. if its *n*th term is not a linear expression in *n*.

1.2.3 General Term of an A.P.

Let *a* be the first term and *d* be the common difference of an A.P. Then its nth term is $a + (n-1)d$ i.e. $a_n = a + (n-1)d$.

1.2.4 nth Term of an A.P. from the end

Let *a* be the first term and *d* be the common difference of an A.P. having *m* terms. Then *n*th term from the end is $(m - n + 1)^{th}$ term from the beginning.

So, nth term from the end = $a + (m-n)d$ and taking a_m as the first term and common difference equal to '$-d$' *n*th term from the end = $a_m + (n-1)(-d)$.

Illustration 1.6 *The first term of an A.P. is - 7 and the common difference 5. Find its 18th term and the general term.*

Solution:

We have, a = first term = - 7 and, d = common difference = 5.

$a_{18} = a + (18 - 1)d$ [$a_n = a + (n-1)d$]

$a_{18} = a + 17d = -7 + 17 \times 5 = 78.$

and, $a_n = a + (n-1) \times 5 = -7 + (n-1) \times 5$

$a_n = -7 + 5n - 5 = \mathbf{5n - 12.}$

Illustration 1.7 *Which term of the A.P. 3, 15, 27, 39,…will be 132 more than its 54th term?*

Solution:

Given A.P. is 3, 15, 27, 39,…

Clearly, its first term = 3 and, common difference = 12.

Let n^{th} term of the A.P. be 132 more than its 54th term

i.e., $a_n = 132 + a_{54}$

$a + (n-1)d = 132 + (a + 53d)$

$3 + 12(n-1) = 132 + (3 + 53 \times 12)$

$12n - 9 = 771$

$12n = 780$

$\boldsymbol{n} = \mathbf{65.}$

Hence, 65th term of the given A.P. is 132 more than its 54th term.

Illustration 1.8 *Divide 69 into three parts which are in A.P. such that the product of the first two is 483.*

Solution:

Since the three parts will be in A.P., let us suppose that the parts are

$a - b, a, a + b$ (i)

∴ by the given condition

$(a - b) + a + (a + b) = 69$ (ii)

and $(a - b)a = 483$ (iii)

Simplifying (ii) we have $3a = 69$ i.e., $a = 23$ (iv)

Substituting (iv) in (iii), we get

$(23 - b)23 = 483$ i.e., $b = 23 - 21 = 2$ (v)

Using (iv) and (v) in (i), the required parts are 23 – 2, 23, 23 +2.

i.e., **21, 23, 25.**

Student Activity

1. The *n*th term of a sequence is given by $a_n = 2n + 7$. Show that it is an A.P. Also, find its 7th term.
2. The *n*th term of a sequence is given by $a_n = 2n^2 + n + 1$. Show that it is not an A.P.
3. Which term of the sequence 4, 9, 14, 19, …is 124?
4. Determine the number of terms in the A.P. 3, 7, 11, …407. Also, find its 20th term from the end.
5. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term and the common difference.
6. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively. Find 32nd term.
7. The sum of 4th and 8th terms of an A.P.is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.
8. The angles of a quadrilateral are in A.P. whose common difference is 10^0. Find the angles.
9. In a garden bed there are 23 rose plants in the first row, twenty one in the second row, nineteen in the third row and so on. There are five plants in the last row. How many rows are there of rose plants?
10. Suba Rao started work in 1995 at an annual salary of ₹ 5000 and received a ₹200 raise each year. In what year did his annual salary will reach ₹7000?

1.2.5 Sum of an Arithmetic Progression

S_n may represent the sum of the n terms of an arithmetic progression, where:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Illustration 1.9 *A sum of ₹280 is to be used to award four prizes. If each prize after the first is 20₹ less than its preceding prize, find the value of each of the prizes.*

Solution:

The value of four prizes form an A.P. with common difference d = - 20, the sum of whose terms is 280.

Let the value of first prize be Rs a. Then,

Sum = 280

$$\frac{4}{2}[2a + (4-1) - 20] = 280$$

$$2(2a - 60) = 280$$

$a - 30 = 70$; $a = 100$.

Hence, the values of 4 prizes are **₹100, ₹80, ₹60 and ₹40**.

Illustration 1.10 *A person is employed in a company at ₹300 per month and is promised an increment of ₹20 per year. Find the total amount which he receives in 25 years and the rate at which he is paid in the last year.*

Solution:

During 1st year he receives at the rate of ₹300 per month, next year at the rate of ₹320 per month, during third year at the rate of ₹340 per month, and so on.

Thus, 300, 320, 340,…, form an A.P. with total number of terms 25 (=n). If Re x per month be the rate at which he is paid in the 25th year of his service, then

$X = a + (n - 1)d$

$= 300 + 24 \times 20 =$ Rs.780.

Total amount = Rs $(300 \times 12 + 320 \times 12 + 340 \times 12 + \ldots + 780 \times 12)$

$= 12 (300 + 320 + 340 + \ldots + 780)$

$= 12 \times \frac{25}{2} (300 + 780)$ [Since, $S = \frac{n}{2} (a + x)$]

$= 12 \times 25 \times 540 =$ **₹1, 62,000**.

Student Activity

11. Find the sum of all odd integers between 2 and 100 divisible by 3.
12. Find the sum of n terms of the series whose nth term is $(2n + 1) 2n$.
13. Find the sum of the series:
    $3 + 5 + 7 + 6 + 9 + 12 + 9 + 13 + 17 + \ldots$ to $3n$ terms.
14. The sum of the first n terms of two A.P.'s are in the ratio $(7n + 2) : (n + 4)$. Find the ratio of their 5th terms.
15. If the 5th and the 12th terms of an A.P. are 30 and 65 respectively, what is the sum of first 20 terms?
16. Find an A.P. in which the sum of any number of terms is always three times the squared number of these terms.
17. A man repays a loan of ₹3250 by paying ₹20 in the first month and then increases the payment by ₹15 every month. How long will it take him to clear the loan?

1.3 INSERTION OF ARITHMETIC MEAN

If between two given quantities *a* and *b* we have to insert *n* quantities A_1, A_2,…,A_n such that a, A_1, A_2, …A_n, b form an A.P., then we say that A_1, A_2,…,A_n are arithmetic means between a and b.

Let *d* be the common difference of this A.P. Clearly, it contains (*n* + 2) terms.

$b = (n + 2)$th term $= a + (n + 1)d$ so, $d = \frac{b-a}{n+1}$

Now, $A_1 = a + d$; $A_2 = a + 2d$; $A_n = a + nd = a + \frac{n(b-a)}{n+1}$

Illustration 1.11 *Insert three arithmetic means between 3 and 19.*

Solution:

Let A_1, A_2, A_3 be 3 A.M.'s between 3 and 19. Then 3, A_1, A_2, A_3, 19 are in A.P. whose common difference is $d = \frac{19-3}{3+1} = 4$

$A_1 = 3 + d = 3 + 4$, $A_1 = 7$; $A_2 = 3 + 2d = 11$; $A_3 = 3 + 3d = 15$.

Hence, the required A.M.'s are **7, 11, 15.**

Illustration 1.12 The sum of two numbers is $\frac{13}{6}$. An even number of arithmetic means are being inserted between them and their sum exceeds their number by 1. Find the number of means inserted.

Solution:

Let *a* and *b* be two numbers such that $a + b = \frac{13}{6}$.

Let A_1, A_2,…,A_{2n} be 2n arithmetic means between a and b.

Then, A_1, A_2,…,A_{2n} = 2n $\left(\frac{a+b}{2}\right)$

$= n(a+b) = \frac{13}{6} n$

It is given that A_1, A_2,…,A_{2n} = 2n + 1

$\therefore \frac{13}{6} n = 2n + 1 \Rightarrow$ **$n = 6$.**

Illustration 1.13 If the A.M. between *p*th and *q*th terms of an A.P. be equal to the A.M. between *r*th and *s*th terms of the A.P., then show that p + q = r + s.

Solution:

Let a be the first term and d be the common difference of the given A.P. Then

a_p = pth term = a + (p-1)d; a_q = qth term = a + (q-1)d

a_r = rth term = a + (r-1)d and a_s = sth term = a + (s-1)d

It is given that

A.M. between a_p and a_q = A.M. between a_r and a_s

$\Rightarrow \frac{1}{2}(a_p + a_q) = \frac{1}{2}(a_r + a_s)$
$\Rightarrow a_p + a_q = a_r + a_s$
$\Rightarrow \{a + (p-1)d\} + \{a + (q-1)d\} = \{a + (r-1)d\} + \{a + (s-1)d\}$
$\Rightarrow (p + q - 2)d = (r + s - 2)d$
$\Rightarrow$ **p + q = r + s.**

Student Activity

18. Insert six A.M.'s between 15 and -13.
19. There are n A.M.'s between 3 and 17. The ratio of the last mean to the first mean is 3:1. Find the value of n.
20. Insert A.M.'s between 7 and 71 in such a way that the 5th A.M. is 27. Find the number of A.M.'s.

1.4 GEOMETRIC PROGRESSION

1.4.1 Geometric Progression

A series in which a ratio of each term, except the first term (which has no preceding terms), to the preceding term is constant throughout is called a **Geometric Progression**. It is commonly written as G.P. The constant ratio is called the **common ratio**.

The terms of G.P. will be such that the ratio of any term to its preceding term is always same. For example, if we consider the sequence 2, 6, 18, 54,…, we observe that the ratio of any element to its preceding element is always 3 and therefore, the sequence is in G.P. Since, the ratio, as stated above, is always the same, it is called the *common ratio*. Thus, if a be the first term and r, the common ratio of a G.P. we may construct the G.P. as

$$a, ar, ar^2, ar^3, \ldots ar^{n-1}, \ldots.$$

If $a, ar, ar^2, ar^3, \ldots ar^{n-1}, \ldots$ be in G.P., find the common ratio.

Let us consider any two terms, say first and second term of the G.P.

Here, $u_1 = a$ and $u_2 = ar$.

Ratio of any term to its preceding term = Any term / Preceding term

= u_2 / u_1

$= ar / a$

$= r.$

The required common ratio = r.

The nature of G.P. finds quite a few applications in annuities, growth of population, depreciated value of assets, etc. where the effect in a particular period of time (say, one year) directly depends upon the size of the commodity at the beginning of the period.

1.4.2 Determination of *nth* term of G.P.

We consider the G.P. $a, ar, ar^2, ar^3 \ldots$ Here, $u_1 = 1^{st}$ term $= a = ar^0 = ar^{1-1}$

$u_2 = 2^{nd}$ term $= ar = ar^1 = ar^{2-1}$

....

From above it is clear that

$$u_n = \text{nth term} = ar^{n-1} \text{ i.e., } \mathbf{u_n = ar^{n-1}}$$

Illustration 1.14 *Find the 8^{th} term of 1, -3, 9, …*

Solution:

Here $a = 1$, $r = -3/1 = -3$, We have to find u_8.

We know $u_n = ar^{n-1}$. $\therefore u_8 = 1 \times (-3)^{8-1} = (-3)^7 =$ **- 2,187.**

Illustration 1.15 *The 4^{th}, 7^{th}, and 10^{th} terms of a G.P. are a, b, c respectively. Show that $b^2 = ac$.*

Solution:

Let the G.P. be $x, xy, xy^2, xy^3, \ldots$..

Then $T_4 = a = xy^3$ (i)

$T_7 = b = xy^6$ (ii)

$T_{10} = c = xy^9$

Now, $b^2 = x^2y^{12}$ [From (ii)]

$= x \times x \times y^3 \times y^9 = x \times y^3 \times x \times y^9$

$= \mathbf{a.c}$ [From (i) and (ii)]

Illustration 1.16 The first term of a G.P. is 1. The sum of the third and fifth terms is 90. Find the common ratio of the G.P.

Solution:

Let r be the common ratio of the G.P. It is given that the first term a = 1.

Now, $a_3 + a_5 = 90$

- $\Rightarrow$ $ar^2 + ar^4 = 90$
- $\Rightarrow$ $r^2 + r^4 = 90 \Rightarrow r^4 + r^2 - 90 = 0 \Rightarrow r^4 + 10r^2 - 9r^2 - 90 = 0$
- $\Rightarrow$ $(r^2 + 10)(r^2 - 9) = 0$
- $\Rightarrow$ $r^2 - 9 = 0 \Rightarrow$ **r = 3.**

Illustration 1.17 The fourth, seventh and the last term of a G.P. are 10, 80 and 2560 respectively. Find the first term and the number of terms in the G.P.

Solution:

Let a be the first term and r be the common ratio of the given G.P. Then,

$a_4 = 10$, $a_7 = 80 \Rightarrow ar^3 = 10$ and $ar^6 = 80$

$ar^6 / ar^3 = 80/10 \Rightarrow r^3 = 8 \Rightarrow$ **r = 2.**

Illustration 1.18 The third term of a G.P. is 4. Find the product of its first five terms.

Solution:

Let a be the first term and r the common ratio. Then,

$A_3 = 4 \Rightarrow ar^2 = 4$

$\therefore$ Product of first five terms $= a_1 a_2 a_3 a_4 a_5$

$= a (ar) (ar^2) (ar^3) (ar^4)$

$= a^5 r^{10} = (ar^2)^5 = \mathbf{4^5}$.

Illustration 1.19 Three numbers are in G.P. whose sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. Find the numbers.

Solution:

Let the numbers be a, ar, ar^2.

Sum = 70 $\Rightarrow a(1 + r + r^2) = 70$ …(i)

It is given that 4a, 5ar, $4ar^2$ are in A.P.

$\therefore 2(5ar) = 4a + 4ar^2$

- $\Rightarrow$ $5r = 2 + 2r^2$
- $\Rightarrow$ $2r^2 - 5r + 2 = 0 \Rightarrow (2r - 1)(r - 2) = 0$
- $\Rightarrow$ $r = 2$ or, $r = ½$.

Putting r = 2 in (i), we obtain a = 10. Putting r = ½ in (i) we get a = 40.

Hence, the numbers are **10, 20, 40 or 40, 20, 10.**

Illustration 1.20 The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 added to its third term, the terms become in A.P. Find the G.P.

Solution:

Let the terms of the given G.P. be $\frac{a}{r}$, a, ar.

Then, product = 1000 => $a^3 = 1000$ => a = 10.

It is given that $\frac{a}{r}$, a + 6, ar + 7 are in A.P.

∴ $2(a + 6) = \frac{a}{r} + ar + 7$

$\Rightarrow$ $32 = \frac{10}{r} + 10r + 7 \Rightarrow 25 = \frac{10}{r} + 10r$

$\Rightarrow$ $5 = \frac{2}{r} + 2r \Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow$ $(2r - 1)(r - 2) = 0 \Rightarrow r = 2.$ ½.

Hence, the G.P. is **5, 10, 20,…. Or 20, 10, 5,…**

Student Activity

21. The seventh term of a G.P. is 8 times the fourth term and 5^{th} term is 48. Find the G.P.
22. If the G.P.'s 5, 10, 20,… and 1280, 640, 320,…have their *n*th terms equal, find the value of n.
23. If 5, *x, y, z*, 405 are the first five terms of a G.P., find the values of *x, y*, and *z*.
24. The sum of three numbers in G.P. is 14. If the first two numbers are each increased by 1 and the third term decreased by 1, the resulting numbers are in A.P. Find the numbers.
25. The product of three numbers in G.P. is 216. If 2, 8, 6 be added to them, the results are in A.P. Find the numbers.
26. The sum of three numbers in G.P. is 56. If 1, 7, 21 are subtracted from the numbers respectively, resulting numbers form the consecutive terms of an A.P. Find the numbers.
27. The pollution in a normal atmosphere is less than 0.01%. Due to leakage of a gas from a factory pollution is increased to 20%. If every day 80% of the pollution is neutralized, in how many days, the atmosphere will be normal?

1.4.3 Sum of *n* Terms of a G.P.

Let a be the first term and r be the common ratio of a G.P. Let S_n be the sum of n terms. Then

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} \qquad \text{(i)}$$

$$rS_n = \quad ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n. \qquad \text{(ii)}$$

[i.e., multiplying both sides by r, and writing the new series below the first so that each term is shifted one place to the right].

Subtracting (ii) from (i), we get

$$S_n (1 - r) = a - ar^n$$

Or $\quad S_n = a\,(1 - r^n) / l - r \quad$ if $r < 1$ $\quad$ (iii)

and $\quad S_n = a\,(r^n - 1) / l - r \quad$ if $r > 1$ $\quad$ (iv)

If l be the last term, then $l = ar^{n-1}$,

$$S_n = lr - a / r - 1$$

Note: It is important to note that we can use (iii) and (iv) for finding S_n according to the value of r.

In case r = 1, then each term of the series is equal to a and ∴

$$S_n = na.$$

Illustration 1.21 *Find the sum of the progression 1, 2, 4, 8,…,256.*

Solution:

Here $a = 1$, $r = 2/1 = 2$, $l = 256$.

Using $\quad S_n = lr - a / r - 1$

$$S_n = 256 \times 2 - 1 / 2 - 1 = 512 - 1 / 1 = \mathbf{512}.$$

Illustration 1.22 Find the sum of 7 terms of the G.P. 3, 6, 12,…

Solution:

Here, a = 3, r = 2.

$\therefore S_7 = a\left(\frac{r^7 - 1}{r-1}\right) = 3\left(\frac{2^7 - 1}{2-1}\right) = 3(128 - 1) = \mathbf{381.}$

Illustration 1.23 *A person has two parents (father, mother), four grand parents, eight great grand parents etc. Find the number of ancestors which a person has in the 12^{th} generation back and total number of all ancestors in these preceding 12 generations, assuming that there are no inter marriages.*

Solution:

Number of parents of 1^{st} generation $\quad = 2$

Number of parents of 2^{nd} generation = 4

Number of parents of 3^{rd} generation = 8

Similarly, number of parents of 4^{th} generation = 16

Thus, we get a series 2, 4, 8, 16,…indicating the number of parents in 1^{st} , 2^{nd} , 3^{rd} , 4^{th} ,… generations respectively.

The series 2, 4, 8,…is a geometric series whose first term $a = 2$ and common ratio $r = 2$.

∴ number of parents of 12^{th} generation = $T_{12} = ar^{11} = 2.2^{11} = 4096$.

Let the number of ancestors in these preceding 12 generations be S_{12}.

But $S_{12} = a(r^{12} - 1) / r - 1$

$= 2(2^{12} - 1) / 2 - 1$

= **8091**.

Student Activity

28. Determine the number of terms in G.P. $<a_n>$, if $a_1 = 3$, $a_n = 96$ and $S_n = 186$.
29. A tennis ball rebound each time to a height equal to one-half of the height of the previous bounce, if it is first dropped from a height of 8 meters, find the total vertical distance it has travelled when it hits the ground for the 10^{th} time.
30. A man borrows ₹8000 at simple interest rate of 2.76% per annum. It is decided that the principal and the interest rate are to be paid in 10 monthly installments. If each installment is double of the preceding installment, find the value of first and the last installment.

1.5 Relationship of Mathematics with other Disciplines

1. ***Social Sciences:*** Disciplines such as economics, sociology, psychology, and linguistics all now make extensive use of mathematical models, using the tools of calculus, probability, and game theory, network theory, often mixed with a healthy dose of computing.

2. ***Economics:*** In economic theory and econometrics, a great deal of mathematical work is being done all over the world. In econometrics, tools of matrices, probability and statistics are used. A great deal of mathematical thinking goes in the task of national economic planning, and a number of mathematical models for planning have been developed. The models may be stochastic or deterministic, linear or non-linear, static or dynamic, continuous or discrete, microscopic or macroscopic and all types of algebraic, differential, difference and integral equations arise for the solution of these models. At a later stage more sophisticated models for international economies, for predicting the results of various economic policies and for optimizing the results are developed.

3. ***Actuarial Science, Insurance and Finance***: Actuaries use mathematics and statistics to make financial sense of the future. For example, if an organization is embarking on a large project, an

actuary may analyze the project, assess the financial risks involved, model the future financial outcomes and advise the organization on the decisions to be made. Much of their work is on pensions, ensuring funds stay solvent long into the future, when current workers have retired. They also work in insurance, setting premiums to match liabilities. Mathematics is also used in many other areas of finance, from banking and trading on the stock market, to producing economic forecasts and making government policy.

4. ***Mathematics in Management:*** Mathematics in management is a great challenge to imaginative minds. It is not meant for the routine thinkers. Different Mathematical models are being used to discuss management problems of hospitals, public health, pollution, educational planning and administration and similar other problems of social decisions. In order to apply mathematics to management, one must know the mathematical techniques and the conditions under which these techniques are applicable. In addition, one must also understand the situations under which these can be applied. In all the problems of management, the basic problem is the maximization or minimization of some objective function, subject to the constraints in available resources in manpower and materials. Thus OR techniques is the most powerful mathematical tool in the field of Management.

5. ***Mathematics in Computers***: An important area of applications of mathematics is in the development of formal mathematical theories related to the development of computer science. Now most applications of Mathematics to science and technology today are via computers. The foundation of computer science is based only on mathematics. It includes, logic, relations, functions, basic set theory, countability and counting arguments, proof techniques, mathematical induction, graph theory, combinatorics, discrete probability, recursion, recurrence relations, and number theory, computer-oriented numerical analysis, Operation Research techniques, modern management techniques like Simulation, Monte Carlo program, Evaluation Research Technique, Critical Path Method, Development of new computer languages, study of Artificial Intelligence, Development of automata theory etc.

Summary

- A series in which the difference obtained by subtracting from any term its immediately preceding term is constant throughout, is called an Arithmetic Progression or arithmetic series. The constant difference obtained in the Arithmetic Progression is called the Common difference

- A series in which a ratio of each term, except the first term to the preceding term is constant throughout is called a Geometric Series and Geometric Progression. The constant ratio is called the common ratio.

Glossary

Arithmetic Progression: A series of quantities form an arithmetic progression if each subsequent term is obtained by adding to the previous term a constant amount.

Geometric Progression: A series of quantities form a geometric progression if each term is obtained by multiplying the previous term by a constant.

Answers to Self Assessment Questions

1. 21
2. Show $a_{n+1} - a_n$ is not independent of n
3. 25^{th} term
4. n =331
5. a =3, d =2
6. 69
7. a = -1/2, d = 5/2
8. 75^0, 85^0, 95^0, 105^0
9. Rows = 10
10. 11
11. S = 867
12. $S = n.2^{n+2} - 2^{n+1} + 2$
13. S = 3n(2n + 3)
14. 5:1
15. S = 1150
16. 3, 9, 15, 21
17. – 65/3 months
18. 11, 7, 3, -1, -5, -9
19. n = 6
20. n = 15
21. 3, 6, 12,…
22. n = 5
23. x = 15, y = 45, z = 135
24. 2, 4, 8 or 8, 4, 2
25. 18, 6, 2 or 2, 6, 18
26. 8, 16, 32 or 32, 16, 8
27. 34.78 days
28. n = 6
29. Distance = 23.969 m
30. 4096

Review Questions

1. The sum of a three digit number is 12. The digits are in arithmetic progression. If the digits are reversed, then the number is diminished by 396. Find the numbers.

2. A machine costing ₹3,200 is bought under an agreement to pay ₹160 per month plus interest on the unpaid balance at the rate of 9%. If the first payment is to be made one month after purchase, find the last payment and the total amount paid.

3. A firm invests ₹50,000 each year in a research programme, where the rate of return is 10% per annum on the investment. What is the total value of the investment after 10 years?

4. Two posts were offered to a man. In the first one the starting salary was ₹2,000 p.m. with the increment of ₹100. In the second post the salary commences at ₹1,500 p.m. but the annual increment was ₹50. The man will accept the post which will give him more earnings in the first 20 years of service, which post should he accept and why?

5. A contractor who fails to complete a building in a certain specified time is compelled to forfeit ₹200 for the first day of extra time required and forfeit thereafter is increased by ₹25 every day, if he loses ₹9,450, for how many days did he over run the contract time?

Further Readings

P.N. Arora, *Mathematics*, S. Chand

R.S. Bhardwaj, *Business Mathematics*, Excel Books.

UNIT 2 INTEREST APPLICATIONS

Unit structure

- Introduction
- Simple Interest
- Compound Interest
- Installment Purchases
- Summary
- Keywords
- Review Questions
- Further Readings

Learning Objectives

After reading this unit you should be able to:

- Computation of Simple interest
- Computation of Compound interest
- Calculation of Effective interest rate
- Calculation of Cost of installment and amortization of loans

2.1 INTRODUCTION

Transactions of money takes place among banks, individuals, business firms and other organizations. In any such transaction there is a lender, who gives money, and a borrower who receives money.

The amount of loan or borrowing involved in the transaction is called the **Principal (P)**.

The borrower pays a certain amount for the use of this money. This is termed as **Interest (I)**.

Interest is always calculated on the principal borrowed.

The borrowing is for a specified period called **Time (t)** and on specified terms.

The specified term is expressed as per cent of the principal and it's called **rate of interest.**

The sum of the principal and the interest is called the **Amount (A)**.

$$A = P + I$$

In general, the rate of interest may be yearly, half yearly, quarterly or monthly as mutually agreed upon by both parties at the time of transaction.

2.2 SIMPLE INTEREST

When the interest is calculated only on the principal initially invested for each year of its use, it is called **simple interest**. In other words, *if the interest is calculated uniformly on the original principal throughout the loan period*, it is called a **simple interest**. Simple interest ***I*** on a principal ***P*** borrowed at the rate of ***r*** per annum (r being in decimal viz., r % = $\frac{r}{100}$ = 0.0r for a period of t years is given by

$$I = \frac{P*R*T}{100}$$

The amount ***(A)*** acquired at the end of ***t*** years is the sum (Principal ***(P)*** and the simple interest ***(I)*** on it.) Thus,

$$A = P + I = P + \frac{P*R*T}{100} = P\left(1 + \frac{rt}{100}\right).$$

A is also known as balance.

Note that for any transaction, the time may be given in months, weeks or days. However, in the simple interest formula, **t** must be in years, and so a conversion must be made:

n months = $\frac{n}{12}$ years	=>	4 months = $\frac{4}{12}$ years
m weeks = $\frac{m}{52}$ years	=>	14 weeks = $\frac{14}{52}$ years
k days = $\frac{k}{365}$ years	=>	63 days = $\frac{63}{365}$ years.

Illustration 2.1 *Find the simple interest on ₹600 at 6% per annum for 5 years. Also find the amount.*

Solution: Here, P = ₹600, r = 6/100; t = 5 years

∴ Interest I = $\frac{P*R*T}{100}$ = ₹ (600 x $\frac{6}{100}$ x 5) = Rs 180.

Amount = Principal + Interest = ₹600 + Rs 180 = ₹**780**.

Illustration 2.2 *How long will it take a sum of money to double itself at 5% simple interest.*

Solution: Let the sum P = ₹ x, then amount A = Rs 2 x

S.I. = ₹2 x – ₹ x = ₹x, r = 5/100 = 0.05

$$I = \frac{P*R*T}{100}$$

$x = x$ x 0.05 x t

t = x / 0.05 x = 100/5 = **20years.**

Illustration 2.3 At what rate per cent annum will a sum of money double in 8 years?

Solution:

Let the sum P = Rs x then amount A = Rs 2 x

t = 8 years, Let r be the rate.

$$A = P\left(1 + \frac{rt}{100}\right)$$

$$2x = x\left(1 + \frac{r X 8}{100}\right)$$

$$2 = P\left(1 + \frac{8r}{100}\right)$$

$$r = \frac{100}{8} = 12 \tfrac{1}{2}$$

r % = **12 ½ %.**

Illustration 2.4 A certain sum of money amounts to ₹1680 in 3 years and to ₹1800 in 5 years. Find the sum and the rate of interest.

Solution:

Amount is given by

A = P + I

P remains the same in both cases. Only amount of interest is different in the two cases because the time periods are different.

P + Interest for 5 years=₹1800

and P + Interest for 3 years=₹1680

On subtraction we get, Interest for 2 years=₹120

∴ Interest for 3 years=₹120x3/2=₹.180

Given : Amount after 3 years = ₹1680

Principal(P) = A – I = ₹(1680 – 180) = ₹ 1500

$$r = \frac{100 X I}{P X t}$$

$$= \frac{100 \ X \ 180}{1500 \ X \ 3}$$

= **4%**

Illustration 2.5 When the bank reduces the rate of interest from 5% to 4% per annum, Ram withdraws ₹500 from his account. If he now gets ₹35 less interest during one year, find how much total money was there in Ram's account initially?

Solution:

Interest on ₹ 500 at 5% for 1 year

$$I = \frac{P \ X \ R \ X \ T}{100}$$

$$I = \frac{500 \ X \ 5 \ X \ 1}{100}$$

I = Rs.25

This ₹ 25 Ram lost even if the rate of interest had not been reduced.

So, the loss of interest due to reduction in the rate of interest = (₹35 - 25) = Rs.10

Reduction in the rate of interest = (5 - 4)%= 1%

i.e. ₹ 1 is lost on every ₹ 100

₹s10 is lost when amount = ₹ 1000

So, Ram's total amount in the account initially = ₹ (1,000+500) = **₹1,500.**

Illustration 2.6 A person has ₹10,000. He invests a part of it at 5% per annum and the rest at 3% per annum simple interest. Thus he earns ₹380 per annum as interest from his total investment. Find the amounts invested in each case.

Solution:

Let the investment at 5% be ₹ x.

Then the investment at 3% is ₹ (10000 - x)

$I = I_1 + I_2$

or, $$380 = \frac{x \ X \ 5 \ X \ 1}{100} + \frac{(10{,}000-x) \ X \ 3 \ X \ 1}{100}$$

$380 \times 100 = 5x + 30000 - 3x$

$2x = 8000$

$x = 4000$

and, $10000 - x = 6000$

So, the investment at 5% is ₹4000 and at 3% it is ₹**6000**.

Illustration 2.7 A person invests a certain sum at a certain rate of simple interest for 4 years. Had he invested it at 2% higher, he would have earned ₹ 240 more. Find the sum he invested.

Solution:

- ₹ 240 is the additional interest in 4 years that he would have earned if he had invested the money at 2% higher rate.

Here we take I = ₹240, t=4 years, r =2%

$$P = \frac{100 \times I}{r \times t}$$

$$P = \frac{100 \times 240}{2 \times 4}$$

P = ₹ **3,000**

Student Activity

1. ₹ 3600 is deposited in a bank for two years at a simple interest of 6%. How much will be the balance at the end of 2 years.
2. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 3% higher rate, it would have fetched ₹300 more? Find the sum.
3. A certain amount of money was invested at 8% simple interest and after 9 months an equal amount was invested at 10% simple interest. Find the period in which the amount in each case becomes ₹52,000. How much money was invested in each case?
4. Amit borrowed ₹830 from Vipul at 12% rate of interest for 3 years. He then added some more money to the borrowed sum and lent it to Pooja for the same time at 14% simple interest. If Amit gains ₹93.90 in the whole transaction, then find

2.3 COMPOUND INTEREST

2.3.1 Compound Interest

If the borrower and the lender agree to fix up a certain interval of time (or period, which may be a year or a half year or a quarter of year etc.), so that the amount (i.e., Principal + Interest) at the end of an interval becomes the principal for the next interval, then the total interest over all the intervals, calculated in this way is called the compound interest. The compound interest is abbreviated as C.I.

Thus,

C.I. = Amount – Principal

2.3.2 Conversion Period

The fixed interval of time (or period) at the end of which the interest is calculated and added to the principal at the beginning of the next interval is called the conversion period.

In other words, a conversion period is the period at the end of which the interest is compounded. If the interest is calculated and added to the principal every six months, then the conversion period is half-yearly, i.e., six monthly. Similarly, the conversion period is quarterly, when the interest is calculated and added after every 3 months.

2.3.3 Calculation of Compound Interest

Let ***P*** be the principal, ***i*** (in % or decimal) as the rate of interest per payment period; ***n*** as the number of payment periods, then accrued amount A_n after ***n*** payments is given by:

$$A_n = P\,(1 + i)^n, \text{where, } i = \frac{\text{annual rate of interest}}{\text{number of payments periods per year}}$$

Also,

(i) If the interest is *compounded annually*, then amount after n years is:

$$A = P\,(1 + r)^n$$

(ii) If the interest is *compounded half yearly*, then amount after n years is:

$$A = P\,(1 + \frac{r}{2})^{2n}$$

(iii) If the interest is *compounded quarterly*, then amount after n years is:

$$A = P\,(1 + \frac{r}{4})^{4n}$$

In general, If the interest is compounded m times a year, then amount after n years is:

$$A = P\,(1 + \frac{r}{m})^{mn}$$

(iv) If the *different rate of interest* is applied in subsequent years:

$$A = P\,(1 + \frac{r1}{100})\,(1 + \frac{r2}{100})\,(1 + \frac{r3}{100})\ldots$$

Where. r_1, r_2, r_3..., are rate of interest for first year, second year, third year and so on.

Compound Interest

$$C.I. = \text{Amount} - \text{Principal}$$

$$= P\left(1 + \frac{r}{100}\right)^n - P$$

$$= P\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$$

Illustration 2.8 Find compound interest on ₹5000 for 2 years at 10% per annum compounded half yearly.

Solution:

Here, P = ₹5000, r = 10% per annum, n = 2 years

For half yearly r becomes 5%, and times becomes 4 years

$$A = P\left(1 + \frac{r}{2 X 100}\right)^n$$

So, $A = 5,000\left(1 + \frac{10}{200}\right)^4 = 5,000\left(1 + \frac{5}{100}\right)^4 = 5,000 \times \frac{1,94,481}{1,60,000} = \frac{1,94,481}{32}$

A = ₹6,077.5

C.I. = ₹ (6,077.5 – 5,000) = **₹1,077.5**

Illustration 2.9 Find compound interest on ₹1000 at 40% per annum, compounded quarterly, for 1 year.

Solution:

Here, P = ₹1000, r = 40% per annum, n = 1 year

For quarterly, r = 40/4 %, n = 1 x 4 years

$$A = P\left(1 + \frac{r}{400}\right)^{4n}$$

So, $A = 1,000\left(1 + \frac{40}{400}\right)^4 = 1,000\left(1 + \frac{10}{100}\right)^4 = 1,000 \times \frac{14,641}{10,000}$

A = ₹1,464.1

C.I. = ₹ (1,464.1 – 1,000) = **₹464.1**

Illustration 2.10 Ram invests ₹5000 in a bond which gives interest at 4% per annum during the first year, 5% during the second year and 10% during the third year. How much does he get at the end of the third year?

Solution:

Here, P = ₹5,000, r_1 = 4%, r_2 = 5%, r_3 = 10%

$$A = P\left(1 + \frac{r1}{100}\right)\left(1 + \frac{r2}{100}\right)\left(1 + \frac{r3}{100}\right)$$

$$A = 5{,}000\left(1 + \frac{4}{100}\right)\left(1 + \frac{5}{100}\right)\left(1 + \frac{10}{100}\right)$$

$= 5{,}000 \times \frac{26}{25} \times \frac{21}{20} \times \frac{11}{10}$ = ₹**6,006**.

Illustration 2.11 A certain sum of money doubles itself in 5 years. In how many years will it become 8 times at the same rate of compound interest?

Solution:

A sum of ₹ x becomes ₹2 x in 5 years

Similarly, ₹2x will become 2 x 2x = ₹4x in next 5 years i.e.; ₹ x will become ₹4$x$in total 10 years.

And ₹4x will become 2 x 4x = ₹8$x$in yet another 5 years.

So, the total time = 5 + 5 + 5 = **15 years**.

Illustration 2.12 The difference between C.I. and S.I. on ₹8000 for 2 years is ₹20. Find the rate of interest.

Solution:

Here, P = ₹8,000, n = 2 years

C.I. - S.I. = Rs 20

$$\therefore \quad P\left(1 + \frac{r}{100}\right)^n - P - \frac{P\,X\,R\,X\,T}{100} = 20$$

$$P\left\{\left(1 + \frac{r}{100}\right)^2 - 1 - \frac{2r}{100}\right\} = 20$$

$$1 + \frac{r2-1}{(100)2} + \frac{2r}{100} - \frac{2r}{100} = \frac{20}{8{,}000}$$

$$\frac{r2}{(100)2} = \frac{1}{400}$$

$$\text{or, } 400 = \left(\frac{100}{r}\right)^2$$

$$r^2 = \left(\frac{100 \times 100}{400}\right) = 25$$

∴ **r = 5%** per annum.

Illustration 2.13 A person opened an account on April, 2013 with a deposit of ₹800. The account paid 6% interest compounded quarterly. On October 1, 2013. He closed the account and added enough additional money to invest in a 6-month Time Deposit for Rs 1000 earning 6% compounded monthly.

(a) How much additional amount did the person invest on October 1?
(b) What was the maturity value of his Time Deposit on April 1, 2014?
(c) How much total interest was earned?

Given that $(1 + i)^n$ is 1.03022500 for i = 1.5%, n = 2 and is 1.03037751 for i = 0.5% and n = 6.

Solution:

(a) The initial investment earned interest for April – June and July – September quarter, i.e. for 2 quarters.

In this case, $i = \frac{6}{4} = 1\frac{1}{2}\%$, n = 2 and the compounded amount = $800\left(1 + 1\frac{1}{2}\%\right)^2$

= 800 x 1.03022500 = ₹824.18

The additional amount = ₹ (1000 – 824.18) = **₹175.82**

(b) In this case, the Time Deposit earned interest compounded monthly for 2 quarters.

Here, $i = \frac{6}{12} = \frac{1}{2}\%$, n = 6, P = 1000

Required maturity value = $1000\left(1 + \frac{1}{2}\%\right)^6$ = 1000 x 1.03037751 = **₹1030.38**

(c) Total interest earned = (24.18 + 30.38) = **₹54.56**

Student Activity

5. Find the compound interest on ₹36,000 for 3 years at 18% per annum, compounded half yearly.
6. The difference between C.I. and S.I. on a certain sum of money at 16% p.a. for 4 years is ₹3,420. Find the sum.
7. The value of a car depreciates at 12% annually. If its present price is ₹1,56,200, what will be its value after 6 years?
8. A sum of ₹5,450 was taken as a loan. This is to be repaid in two equal annual installments. If the rate of interest be 18% p.a., compounded annually, find the value of each installment.
9. Due to migration to the cities, the population of a village decreases at the rate of 4% p.a. If its present population is 2,540, what will it be after 5 years.

2.3.4 Effective rate of interest

It is equivalent annual rate of interest due to compounding. If interest is compounded more than once a year the effective rate exceeds the per annum rate but there does not exist difference between the above two rates, in case, interest is compounded annually. The effective interest rate, E, is computed from :

$E = (1 + r)^n - 1$, where *r* and *n* are as defined earlier. The number 1 in the expression $P(1 + i)^n$ is due to principal and consequently, we subtracted 1 from the sum to find the interest only.

Illustration 2.14 Let ₹100 be invested for a year at 10% rate of interest per annum compounded half yearly then the amount A after 1 year is

$$A = 100\left(1 + \frac{10}{200}\right)^2 = 100\left(\frac{21}{20}\right)^2 = 110.25$$

∴ Interest earned = ₹110.25 – ₹100 = ₹10.25

Thus, the actual interest earned on ₹100 is ₹10.25, which represents an annual return of 10.25%. We say that the effective rate of interest in this case is 10.25%, whereas the nominal rate of interest is 10%.

2.3.5 Force of Interest

The nominal rate *r* compounded continuously and the equivalent effective rate e_r is called the force of interest.

Illustration 2.15 Which is better investment: 9% per annum compounded quarterly or 9.1% per year simple interest?

Solution:

Effective rate corresponding to 9% compounded quarterly is

$$r_e = \left(1 + \frac{0.09}{4}\right)^4 - 1 = (1.0225)^4 - 1$$

$$= 1.09308 - 1 = 0.931 \text{ or } \mathbf{9.31\%}$$

Since 9.31% is more than 9.1% per year simple interest, so the first investment is the better investment.

Illustration 2.16 Which yields more interest: 7.8% compounded semi-annually or 8% compounded quarterly?

Solution:

The effective rate corresponding to 7.8% compounded semi-annually

$$r_e = \left(1 + \frac{7.8}{200}\right)^2 - 1 = (1.039)^2 - 1$$

$$= 1.0759 - 1 = .0759 \text{ i.e., } 7.95\%$$

The effective corresponding to 8% compounded quarterly is

$$r_e = \left(1 + \frac{8}{400}\right)^4 - 1 = (1.02)^4 - 1 = .0824 \text{ i.e., } 8.24\%$$

Since the effective rate corresponding to **8%** compounded quarterly is more than the effective rate corresponding to **7.8%** compounded semiannually, therefore, the second choice is preferable.

2.4 ANNUITY AND INSTALLMENT

2.4.1 Annuity

An annuity is a series of regular payments of a fixed sum of money, usually equal in size, and made at equal intervals of time. These equal intervals may a year, a half-year, a quarter, a month or a day. In other words, an annuity is an installment. For example; repayment installments of loan taken for house construction or of loan for purchase of a car; L.I.C. premiums; deposits into the recurring account etc., are all examples of annuities.

Periodic Payments*: The size of each payment of an annuity is called the periodic payment or periodic rent of the annuity.*

Payment Period: *The time between two successive payment dates of an annuity is called the periodic period or payment interval.*

Term of an Annuity: *The total time from the beginning of the first payment period to the end of the last payment period is called the term of the annuity.*

Present value of an Annuity: *The present value of an annuity is the sum of all the present values of its installments.*

Amount of an Annuity: *The amount or future value of an annuity is the value, at the end of the term, of all installments.* In other words, the total worth of all the payments at the conclusion of the annuity is called the amount of annuity or future value of the annuity.

> Amount of Annuity = [Sum of all installments + Compound interest earned on the installments in the end of the term of the annuity].

2.4.2 Instalment Buying

In the instalment buying scheme a customer is not required to make full payment of the article at the time of buying it but is allowed to pay a part of it at the time of purchase and rest in easy instalments, which could be monthly, quarterly, half yearly or even yearly. *The time between two successive instalment dates is called* **payment period**. The instalments are generally paid at the end of the period. Thus, if the instalments are monthly, then first instalment is paid at the end of first month, second at the end of second month and so on. In instalment buying (purchase) scheme the buyer will have to pay more than the actual price of the article, because the seller charges some interest for deferred payments (payments made at later dates).

Terms:

(a) With **instalment buying** you repay a loan on a monthly basis and you get charged interest. The interest worked into the monthly price is known as a **finance charge**, which is worked into the monthly price. Here you have the advantage that you get to have the product right away, even though you haven't fully paid for it.

(b) The **cash price** is the amount of the item you want to buy. The **amount you finance** *is the total you borrow*. The **down payment** *is the amount of money you pay right away*. They are related by the following relationship:

Amount Financed = Cash Price - Down Payment

(c) The **total instalment price** is the total amount you pay i.e. all monthly payments plus down payment.

Total Installment Price = (Monthly payment) x (Number of payments) + Down payment

(d) The **finance charge** is the amount you pay for borrowing the money (the interest paid)

Finance Charge = Total Installment Price - Cash Price

(e) The **future value of an instalment** is the sum of the value of an instalment and interest earned on it at the end of the instalment scheme.

(f) The **future value of all instalments** is the sum of the future values of all the instalments at the end of the instalment scheme.

These terms will be better understood with the help of the following illustrations:

Illustration 2.17 *The cost of a new car is ₹15,00,000. You can make down payment of ₹3,00,000 and finance the rest for ₹31,500 per month for 60 months. Find the amount financed, total installment price and finance charge.*

Solution:

Amount Financed = ₹15,00,000 – ₹3,00,000 = **₹12,00,000**

Total Installment Price = (31,500) 60 + ₹3,00,000 = **₹21,90,000**

Finance Charge = ₹21,90,000 - ₹15,00,000 = **₹6,90,000**

2.4.3 Effective Interest Rate in Case of Loans

The *effective annual interest rate* on loans measures the real (true) cost of credit. The calculation of the effective interest rate varies depending on the type of loan. Let us looks at various types of loans to see how the effective interest rate is determined.

Effective annual interest rate on single-payment loans

Single-payment loans are the loans that are paid off on a given date. There are two methods that can be used to calculate the effective annual interest rate on single-payment loans (refer to the table 1 below).

Table 1: Effective annual interest rates (EAR) on single-payment loans

<table>
<tr><td rowspan="2">Simple interest method</td><td>Interest is calculated on the amount borrowed.
Stated and effective interest rates are the same.</td></tr>
<tr><td>EAR = Interest ÷ Proceeds</td></tr>
<tr><td colspan="2">When a loan requires a compensating balance:
Proceeds = Principle x Proceeds Percentage
Proceeds Percentage =100% - Compensating Balance Percentage</td></tr>
<tr><td rowspan="2">Discount method</td><td>Interest is deducted from the amount of the loan.
Borrower prepays the finance charge.</td></tr>
<tr><td>EAR = Interest ÷ (Principal – Interest)</td></tr>
<tr><td colspan="2">When a loan requires a compensating balance:
Proceeds = Principal x Proceeds Percentage - Interest
Proceeds percentage =100% - Compensating Balance Percentage</td></tr>
</table>

Illustration 2.18 Company ABC can take 1,00,000 loan from Bank X or Bank Y. Bank X can give the one-year loan at 10% paid at maturity, while Bank Y can lend on a discounted basis at a 10% interest rate. Which bank charges a lower effective annual interest rate (EAR)?

Solution:

EAR (Bank X) = 0.10

EAR (Bank Y) = $\frac{1,00,000\ X\ 0.10}{1,00,000\ X\ (1-0.10)}$ = **0.1111**

As we can see, Company ABC should take the loan from Bank X as it charges the lower EAR.

Illustration 2.19 *Company ABC took Rs 500,000 loan at a 10% interest rate from Bank X. The interest is due at maturity. Bank X requires a 20% compensation balance. What is the effective annual interest rate?*

Solution:

EAR = $\frac{\text{Principal X Interest rate}}{\text{Principal X (1} - \text{compensating balance \%)}}$

EAR = $\frac{5,00,000\ X\ 0.10}{5,00,000\ X\ (1-0.20)}$ = **0.1250**

2.4.3(a) Effective annual interest rate on a line of credit

Under a line of credit, a borrower can lend money from a lending institution (e.g. bank) on a recurring basis up to a certain amount. The borrower might be required to maintain a deposit that does not earn interest. Such a deposit is called a **compensating balance** that is usually presented as a percentage of the loan. If a compensating balance is placed on the unused portion of the line of credit, the interest rate will be smaller.

To calculate the effective annual interest rate on a line of credit, the following formula can be used:

$$\textbf{EAR} = \frac{\text{Principal X Interest rate}}{\text{Principal} - \text{Compensating balance}}$$

Illustration 2.20 Company ABC has ₹1,00,000 line of credit at Bank Z. The company must maintain the following compensating balances: 15% on outstanding loan and 10% on the unused portion of the line of credit. The company borrowed ₹ 60,000. The interest rate on the loan is 12%. What is the effective annual interest rate?

Solution:

The required deposit equals: 60,000 x 15% + 40,000 x 10% = 13,000

EAR = $\frac{60,000\ X\ 0.12}{60,000 - 13,000}$ = 0.1532

The effective interest rate on this loan (on a line of credit) is **15.32%**

2.4.3(b) Effective annual interest rate on installment loans

An installment (amortized) loan is a loan that is periodically paid off in equal installments. Examples may include car loans, commercial loans, and mortgages.

There are four methods used to calculate the effective annual interest rate on installment loans (refer to the table 2 below).

Table 2: Effective interest rates on installment loans

Actuarial method	• Most accurate method • Used by lenders • Complicated formulas
Constant-ratio method	• Simple formula • Overstated EAR • Higher quoted rate, more overstated EAR
	EAR = $\frac{2 \times M \times C}{P \times (N+1)}$ ***M*** is the number of payment periods per year ***C*** is the cost of credit (finance charges) ***P*** is the original proceeds **N** is the number of scheduled payments
Direct-ratio method	• Simple formula • More complicated than constant-ratio method but less complicated than actuarial method • Slightly understates effective annual interest rate
	EAR = $\frac{6 \times M \times C}{3 \times P \times (N+1) + C \times (N+1)}$ ***M*** is the number of payment periods per year ***C*** is the cost of credit (finance charges) ***P*** is the original proceeds ***N*** is the number of scheduled payments
N-ratio method	• More accurate than constant-ratio or direct-ratio methods • Effective annual interest rate is slightly overstated or understated

	depending on the nominal rate and the maturity of the loan
	EAR = $\frac{M\,X\,C\,X\,(95\,X\,N+9)}{12\,X\,N\,X\,(N+1)X\,4P+C)}$ ***M*** is the number of payment periods per year ***C*** is the cost of credit (finance charges) ***P*** is the original proceeds ***N*** is the number of scheduled payments

If the amount of payment or time between payments varies from period to period (e.g., balloon payments), the constant-ratio, direct-ratio, and N-ratio methods cannot be used. If a lender charges a credit investigation, loan application, or life insurance fee, such a cost should be added to the cost of credit (finance charge).

Illustration 2.21 Company ABC borrows ₹ 12,000 to be repaid in 12 months. The monthly installments are ₹,116 each. The finance charge is ₹,400. What is the approximate value of effective annual interest rate?

Solution:

Constant ratio method:

$$\text{EAR} = \frac{2\,X\,12\,X\,1400}{12{,}000\,X\,(12+1)} = 0.2154$$

Direct-ratio method:

$$\text{EAR} = \frac{6\,X\,12\,X\,1400}{3\,X\,12{,}000\,x\,(12+1)+\,1400\,X\,(12+1)} = 0.2073$$

N-ratio method:

$$\text{EAR} = \frac{12\,X\,1400\,(95\,X\,12+9)}{12\,X\,12\,(12+1)(4\,X\,12{,}000+1400)} = 0.2104$$

Using the actuarial method, the effective annual interest rate is likely to be close to **21.04%.**

As we can see from this example, the constant-ratio method overstated the effective annual interest rate, while the direct-ratio method slightly understated the effective annual interest rate on the installment loan.

There is another method used to approximate this rate on one-year installment loans to be paid in equal monthly installments. *The effective interest rate is determined by dividing the interest by the average amount outstanding for the year.* If the loan is discounted, the average loan balance equals the average of proceeds (i.e., principal less interest).

Illustration 2.22 *Company ABC borrows ₹10,000 at a 10% interest rate to be paid in 12 monthly installments.*

Solution:

EAR could be approximated as follows:

Interest = ₹12,000 x 0.10 = ₹1,200

Average Loan Balance = ₹$\frac{12,000}{2}$ = ₹6,000

Effective Annual Interest Rate = ₹$\frac{1,200}{6,000}$ = 0.20

If this loan is discounted, the effective annual interest rate will be calculated as follows:

Interest = ₹1,200

Proceeds = ₹12,000 - ₹1,200 = ₹10,800

Average Loan Balance: ₹$\frac{10,800}{2}$ = ₹5,400

Effective Annual Interest Rate = ₹$\frac{1,200}{5,400}$ = **0.2222**

2.4.4 Amortization of a Loan

In lending, **amortization** is the distribution of payment into multiple cash flow installments, as determined by an amortization schedule. Unlike other repayment models, each repayment installment consists of both principal and interest. Amortization is chiefly used in loan repayments (a common example being a mortgage loan) and in sinking funds. Payments are divided into equal amounts for the duration of the loan, making it the simplest repayment model. A greater amount of the payment is applied to interest at the beginning of the amortization schedule, while more money is applied to principal at the end. Commonly it is known as EMI or Equated Monthly Installment

$$P = A \cdot \frac{1-\left(\frac{1}{1+r}\right)^{n}}{r}$$

or, equivalently,

$$A = P \cdot \frac{r(1+r)^{n}}{(1+r)^{n}-1}$$

where: *P* is the principal amount borrowed, *A* is the periodic amortization payment, *r* is the periodic interest rate divided by 100 (nominal annual interest rate also divided by 12 in case of monthly installments), and *n* is the total number of payments (for a 30-year loan with monthly payments $n = 30 \times 12 = 360$).

Negative amortization (also called deferred interest) occurs if the payments made do not cover the interest due. The remaining interest owed is added to the outstanding loan balance, making it larger than the original loan amount.

If the repayment model for a loan is "fully amortized," then the very last payment (which, if the schedule was calculated correctly, should be equal to all others) pays off all remaining principal and interest on the loan. If the repayment model on a loan is not fully amortized, then the last payment due may be a large balloon payment of all remaining principal and interest. If the borrower lacks the funds or assets to immediately make that payment, or adequate credit to refinance the balance into a new loan, the borrower may end up in default.

With installment buying you repay a loan on a monthly basis. You get charged interest, known as a finance charge, which is worked into the monthly price. The advantage is that you get to have the product right away, even though you haven't completely paid for it.

The cash price is the amount of the item you want to buy. The amount you finance is the total you borrow. The down payment is the amount of money you pay right away. They are related by the equation below.

Amount Financed = Cash Price – Down Payment

The total installment price is the total amount you pay (all monthly payments plus down payment)

Total Installment Price = (Monthly payment) × (Number of payments) + Down payment

The finance charge is the amount you pay for borrowing the money (the interest paid)

Finance Charge = Total Installment Price – Cash Price

Illustration 2.23 The cost of a new car is ₹1,40,000. You can pay ₹2,800 down and finance the rest for ₹3,150 per month for 60 months. Find the amount financed, total installment price and finance charge.

Solution:

Amount Financed =₹1,40,000 – ₹2,800 = **₹1,37,200**

Total Installment Price = (₹3,150) 60 + ₹2,800 = **₹1,91,800**

Finance Charge =₹1,91,800 – ₹1,40,000 = **₹51,800**

The Formula Involved in Borrowing:
The interest rate per year is called the Annual Percentage Rate (APR), and lenders are required by law to inform you of the APR on any loan.

$A = (1-1 \div (1+i)^n) \times R \div i$ and

$R = A \times i \div (1-1 \div (1+i)^n)$ relate the quantities

A = amount borrowed
R = monthly payment
i = monthly interest rate (APR/12), and
n = total number of payments

These formulas will prove to be useful in two cases. 1) you can compute the monthly payment on a given loan, or 2) you can compute the amount of money paid in finance charges.

Illustration 2.24 If you purchase a truck for ₹9,00,000 with no money down at 0.9% per month for 60 months what is your monthly payment?

Solution:

$A = 9,00,000$, $i = 0.009$, and $n = 60$

$R = A \times i \div (1-1\div (1+ i)n) = 9,00,000 \times 0.009 \div (1-1\div (1+ 0.009)60) = 194.79$

You will owe ₹**194.79** per month

In the above example, what is amount paid in finance charges?

Solution:

Amount Paid = 194.79 (60) = ₹11,687.40

Finance Charge = 11,687.40 – 9,000 = ₹**2,687.40**

Summary:

- When money is borrowed, interest is charged for the use of that money for a certain period of time. When the money is paid back, the principal (amount of money that was borrowed) and the interest is paid back. The amount to interest depends on the interest rate, the amount of money borrowed (principal) and the length of time that the money is borrowed.
- Interest is calculated on the initial principal and also on the accumulated interest of previous periods of a deposit or loan. Compound interest can be thought of as “interest on interest,” and will make a deposit or loan grow at a faster rate than simple interest, which is interest calculated only on the principal amount. The rate at which compound interest accrues depends on the frequency of compounding; the higher the number of compounding periods, the greater the compound interest.
- *A loan with scheduled periodic payments of both principal and interest.* This is opposed to loans with interest-only payment features, balloon payment features and even negatively amortizing payment features.

Glossary:

Simple Interest: If the interest on a certain sum borrowed for a certain period is calculated uniformly, it is called simple interest i.e., the interest is calculated only on the principal borrowed.

Compound Interest: Compound interest arises when interest is added to the principal of a deposit or loan, so that, from that moment on, the interest that has been added also earns interest. This addition of interest to the principal is called compounding.

Amortized loan: An amortizing loan is a loan where the principal of the loan is paid down over the life of the loan (that is, amortized) according to some amortization schedule, typically through equal payments.

Answers to Self Assessment Questions

1. ₹ 4032
2. S = 5000
3. N = 3years, 9 months; Rs 40,000
4. S = 935
5. ₹ 2,42,360
6. S = 20,000
7. ₹ 72,560
8. ₹ 3,481
9. 2071

Review Questions

1. Mr. Anurag Awasthi deposits an amount of ₹ 56,500 to obtain a simple interest at the rate of 12% p.a. for 3 years. What total amount will Mr. Anurag Awasthi get at the end of the year?

2. The simple interest accrued on an amount of ₹1,98,00 at the end of three years is ₹7,128. What would be the compound interest accrued on the same amount at the same rate in the same period?

3. Mr. Sane invested a total amount of ₹1,65,00 for two years in two schemes A and B with rate of simple Interest 10% p.a. and 12% p.a. respectively. If the total amount of interest earned was ₹3,620, what was the amount invested in Scheme B?

4. A sum of ₹6,630 is divided into two parts in such a way the first part after 9 years and the second part after 11 years amount to the same value. The rate of interest is 10% p.a., compounded annually. Find each part.

5. The difference between compound interest and simple interest on a certain sum at 10% p.a. for 4 years is ₹1,282. Find the sum.

6. A housing loan of ₹4,00,000 was to be repaid over 20 years by monthly installments of an annuity-immediate at the nominal rate of 5% per year. After the 24th payment was made, the bank increased the interest rate to 5.5%. If the lender was required to repay the loan within the same period, how much would be the increase in the monthly installment. If the installment remained unchanged, how much longer would it take to pay back the loan?

7. A housing loan is to be repaid with a 15-year monthly annuity-immediate of ₹2,000 at a nominal rate of 6% per year. After 20 payments, the borrower requests for the installments to be stopped for 12 months. Calculate the revised installment when the borrower starts to pay back again, so that the loan period remains unchanged. What is the difference in the interest paid due to the temporary stoppage of installments?

8. A purchased a TV system for ₹15,000 and agrees to pay it by monthly payments of ₹750. If the store charges interest at the rate of 12 per cent compounded monthly, how many months will it take to pay off the debt?

Further Readings

P.N. Arora, *Mathematics*, S. Chand

R.S. Bhardwaj, *Business Mathematics*, Excel Books.

T.R. Jain, *Business Mathematics*, VK Enterprises

UNIT 3 PERCENTAGE AND COMMISSION

Unit structure

- Introduction
- Percent
- Commissions
- Summary
- Keywords
- Review Questions
- Further Readings

Learning Objectives

After reading this unit you should be able to:

- Rules of percentages
- Use of percentages
- Computation of Commission

3.1 INTRODUCTION

Percentage is a mathematical concept that appears very frequently in everyday life. You read that a merchant is offering a twenty per cent discount on a selected group of items. The manufacturer of an article of clothing states that the material is sixty five per cent dacron and thirty five per cent polyester. Savings banks pay a Five and one half per cent (5.5%) interest rate annually on regular savings accounts. The state of Punjab has raised the rate of its sales tax to seven and one half percent (7.5%).

These and similar items which appear daily in newspapers indicate the importance of understanding the concept of percentage. In order to live more insightfully, wisely and enjoyably every citizen should be able to perform simple mathematical calculations which enable him to compute discount and sales tax on purchases, gratuities for services performed, interest on savings accounts and loans, deductions from his monthly pay and other consumer related problems which he encounters from day to day.

3.2 PERCENTAGE

Percentage is a method of expressing fractions or parts of any quantity into an equivalent form. Percentages are expressed in terms of hundredths. It is denoted by the symbol %.

So, 100% means 100 hundredths or 100/100 = 1

50% means 50 hundredths or 50/100 = ½

In other word, the rate per cent is the quantity for every hundred.

Percentage – Decimal Conversion

When we say that Ram spends 25% of his income on food, it means for every ₹100 earned Ram spends ₹25 on his food.

Percentage can be expressed as a fraction with numerator equal to rate per cent and denominator as 100. Its decimal equivalent can be easily written by putting in the rate per cent a decimal point after leaving two extreme right digits.

For example, 125% = 1.25

25% = 0.25

5% = 0.05 and so on.

In order to convert a decimal into a percentage we need to multiply the decimal number by 100. This is equivalent in moving the decimal point by two digits to its right.

For example, 0.145 = 14.5%

0.54 = 54%

0.3 = 30%

0.03 = 3% and so on.

Percentage – Fraction Conversion

A fraction is converted into its percentage form by first changing the fraction into a decimal number and then changing the decimal into a percentage. Some cases where denominator is a submultiple or multiple of 100, we reduce the denominator to 100 by multiplying with a suitable factor both its numerator and denominator.

For example, $\frac{2}{5} = (\frac{2}{5} \times 100) = 40\%$

$\frac{20}{400} = \frac{5}{100} = 5\%$

To convert a percentage into a fraction, we write the rate percent in the numerator and put 100 in the denominator. We then reduce it to its lowest terms.

For example, $45\% = \frac{45}{100} = \frac{9}{20}$

$125\% = \frac{125}{100} = \frac{5}{4}$

$33\% = \frac{33}{100}$ and so on.

Illustration 3.1 A man pays 12% income tax. If his annual income is ₹75,000, how much does he pay as income tax annually?

Solution:

Annual Income of man = ₹75,000

Income Tax = 12%

Total Income Tax paid by him = 12% of ₹75,000 = $\frac{12}{100} \times 75{,}000$

$= \frac{3}{25} \times 75{,}000 =$ ₹**9,000.**

Illustration 3.2 The monthly salary of an employee is ₹6000. He spends 20% on the education of his children, 25% on house rent, 5% on travels, 30% on food, 8% on miscellaneous items and rest he saves for the future. Find his savings.

Solution:

Monthly salary = ₹6,000

His total expenditure = 20% + 25% + 5% + 30% + 8% = 88%

Thus, his savings = (100 – 88)% = 12%

His savings per month = 12% of ₹6000 = $\frac{12}{100}$ x 6000

= **₹720.**

Illustration 3.3 The food grain production during eighth plan increased from 180 million to 190 million tonnes. Find the percentage increase in production.

Solution:

Total Increase = 190 – 180 = 10 million tones

Percentage increase in production = $\frac{10}{180}$ x 100 = **5.56%**

Illustration 3.4 Ram makes a profit of ₹ 12,000 in a business. 60% of the profit he reinvests in his business. Of the remaining profit 30% he distributes as bonus to his employees, 40% on charity and rest on business promotion. Find the distribution of his profits under various heads.

Solution:

Ram's profit = ₹12,000

Reinvestment in business = 60%

i.e., Reinvestment = 60% of ₹12,000

= $\frac{60}{100}$ x 12000 = ₹7,200

Amount remaining for expenditure = ₹12,000 – ₹7,200 = ₹4,800

Alternatively, (100 – 60)% of ₹12,000

= 40% of ₹12,000 = ₹4,800

Bonus = 30% of ₹4,800 = $\frac{30}{100}$ x 4,800 = ₹1,440

He also contribute 40% of remaining profit towards charity

Amount contributed as charity = 40% of ₹4,800 = $\frac{40}{100}$ x 4,800 = ₹1,920

Amount left for Business promotion = ₹4,800 – (₹1,440 + ₹1,920)

= ₹1,440

or, 100% - 30% - 40% = 30%

and, 30% of Rs 4,800 = **₹1,440.**

Illustration 3.5 Due to better medical care the death rate has decreased by 5% to 3.5 per thousand. What was the original death rate?

Solution:

Let the original death rate be x

5% decrease means the present death rate is (100 – 5) = 95% of the original death rate

95% of x = 3.5

$x = \frac{3.5}{0.95}$; $\mathbf{x = 3.68}$

Rules of percentage

a. *If the original population of a town is* **P** *and it increases at the rate of* **r** *% per annum*,

Population after ***n*** years will be $\mathbf{P(1 + \frac{r}{100})^n}$

Population ***n*** years ago was $\mathbf{P \div (1 + \frac{r}{100})^n}$

If the population decreases at the rate of **r** *% per annum*,

Population after ***n*** years will be $\mathbf{P(1 - \frac{r}{100})^n}$

Population ***n*** years ago was $\mathbf{P \div (1 - \frac{r}{100})^n}$

b. *To increase a given number by a given* **r** *%, multiply the given number by the factor* $\frac{100+r}{100}$ *and to decrease replace plus sign in the factor i.e.*, $\frac{100-r}{100}$.

c. *If A and B are respectively* ***a****% and* ***b****% greater than the third quantity C, then*

$$\textbf{A} = \frac{100+a}{100+b} \textbf{ x 100\% of B}$$

d. *If A is **a**% of C and B is **b**% of C, then*

$$\textbf{A} = \frac{a}{b} \textbf{ x 100\% of B}$$

e. (i) *There is a certain initial quantity x. If **a**% of **x** is taken out first, then **b**% of the remainder is taken out on the second time, then **c**% of the remainder on the third occasion, and the balance left is **y**, then we have the relation*

$$\textbf{X} = \frac{100}{100-a} \textbf{ x } \frac{100}{100-b} \textbf{ x } \frac{100}{100-c} \textbf{ y}$$

(ii) *If instead of taking out, it is a case of adding, we simply replace minus sign by plus sign i.e.,*

$$\textbf{X} = \frac{100}{100+a} \textbf{ x } \frac{100}{100+b} \textbf{ x } \frac{100}{100+c} \textbf{ y}$$

f. *If A is **r**% more than B, then B is ($\frac{r}{100+r}$ x 100)% less than A*

$$\textbf{A – B} = (\frac{r}{100+r} \textbf{ x 100)\% of A}$$

*Similarly, if A is **r**% less than B, then B is more than A*

$$\textbf{B – A} = (\frac{r}{100-r} \textbf{ x 100)\%}$$

g. *If a is first increased by x % and then decreased by **y**%*

$$\textbf{Final value} = A(\frac{100+x}{100}) \textbf{ x } (\frac{100+y}{100})$$

$$\textbf{Net \% change} = [x + y + \frac{xy}{100}]\%$$

Illustration 3.6 If Mohan’s salary is 50% more than Shyam’s salary, by how much percent is Shyam’s salary less than Mohan’s salary?

Solution:

Let the required percentage = x

Mohan’s salary = 50% more than Shyam’s salary

By using x = $(\frac{r}{100+r}$ x 100)% = $(\frac{50}{100+50}$ x 100)%

$= \frac{100}{3}$ % = **$33\frac{1}{3}$%**

Illustration 3.7 The price of TV was reduced twice by 40% and 10% respectively. Find the net percentage decrease in price.

Solution:

Net % change = [x + y $\frac{xy}{100}$+]%

= [- 40 - 10 + $\frac{(-40)(-10)}{100}$]%

= (- 50 + 4)%

Net % change = - 46%

or, Net % decrease = **46%** (Negative sign shows decrease in percentage)

Illustration 3.8 The population of a city is 12 lakhs. Its population increases at the rate of 3% per annum. What was its population 3 years ago and also what will be its population after 3 years?

Solution:

Population (P) = Rs 12 lakhs

Rate of increase of population = 3%

∴ Population *n* years ago = P ÷ $(1 + \frac{r}{100})^n$

So, population 3 years ago = 12 ÷ $(1 + \frac{3}{100})^3$

$= \frac{12}{(1.03)(1.03)(1.03)}$ lakhs = 10.9817 lakhs = 10,98,170

Population after *n* years will be P$(1 + \frac{r}{100})^n$

So, Population after 3 years = $12(1 + \frac{3}{100})^{3}$ s

$= 12\,(1.03)^{3} =$ **13.11272 lakhs.**

Illustration 3.9 Ram saves 20% of his salary but due to increase in prices of commodities his expenses increase by 20%. Still he saves ₹20. Find his salary.

Solution:

Let Ram's salary be ₹x

Initial saving = $0.20x$

Initial expenses = $x - 0.20x = 0.80\,x$

Increase in expenditure = $0.20 \times 0.80x = 0.16\,x$

Final expenditure = $0.80x + 0.16\,x = 0.96\,x$

Final saving = $x - 0.96x = 0.04\,x$

$0.04x =$ ₹20

$x = \frac{20}{0.04} =$ ₹500 ∴ Ram's salary is **₹500.**

Illustration 3.10 After 10% reduction in the price of pen, a student buys 4 more pens in ₹160. Find the original and the new price of the pen.

Solution:

Let the original price of the pen be ₹ x

Reduction in price of pen = 10%

∴ New price = ₹ ($x - 0.10\,x$) = ₹0.90 x

Saving on ₹160 = 10% of ₹160 = ₹16

New price of 4 pens = 4 X ₹0.90 x = ₹3.6 x

These additional 4 pens are bought out of savings due to reduction in price of the pen

So, $3.6x = 16$

$$\Rightarrow \quad x = \frac{16}{3.6}$$

$$\Rightarrow \quad x = \frac{160}{36} = \frac{40}{9}$$

Old price of a pen = ₹40/9

New price of a pen = ₹ (0.9 x 40 ÷ 9) = **₹4.**

Illustration 3.11 A's earning is 10% more than C's. B's earning is 15% more than C's. Find what percent is A's earning of B's?

Solution:

Let 'a' be the A's earning more than C's earnings

Let 'b' be the B's earning more than C's earnings

i.e. a = 10% and b = 15%

Then, relation between A's and B's earnings is given by:

$$A = \frac{100+a}{100+b} \text{ x } 100\% \text{ of B}$$

$$A = \frac{100+10}{100+15} \text{ x } 100\% \text{ of B}$$

$$A = \frac{110}{115} \text{ x } 100\% \text{ of B}$$

A = **95.65% of B.**

Student Activity

1. The price of LPG is increased by 20%. Find by how much per cent its consumption must be increased so as not to decrease the expenditure?
2. Ram gets a lump sum amount on retirement. First he spends 60% to buy a house. 40% of the remainder he spends on his daughter's marriage. From the balance he invests 70% in a business and finally he is left with ₹21,600. How much did he get on retirement?
3. At an election there are two candidates in the contest. The candidate who gets 62% of the votes is declared elected by a margin of 144 votes. Find the total number of votes polled. Assume that the none of the votes polled was recorded invalid.
4. In an examination full marks was 500. A got 10% less than B's marks, B got 25% more than C's marks and C got 20% less than D's marks. If A got 360 marks, what % did D get?
5. A sum of ₹817 is divided among A, B and C such that 'A' receives 25% more than 'B' and 'B' receives 25% less than 'C'. What is the 'A' share in the amount?
6. Shilpa spent 8% on school fees, 25% on rent and 17% on furniture. 25% of the remaning amount was spent on medical bills and the remaining ₹6,000 was set aside for investment. How much money does she spend on rent?
7. The population of a State is counted after every three years. It is found that the population each time is increased by 20% as compared to the previous count. If the population in the year 2003 was 42 lakhs, what will be the population in the year 2012 in lakhs?

s

3.3 COMMISSION

Manufacturers and producers of goods are not always able to sell their own products. It is necessary for them to employ agents to sell the articles for them. *The pay received by the agent, or salesman, for work or services performed is called* ***commission***. Sometimes the commission is a certain amount for each article sold. Other times it is a percentage of the dollar value of the sales. *That rate of percent is called the* ***rate of commission***.

The total money received by the salesman for his employer is called the ***gross proceeds****. The gross proceeds minus the commission is known as the* ***net proceeds****.*

Net Proceeds = Gross Proceeds - Commissions

Sometimes a salesman receives a graduated commission. This means that *the rate of commission increases as the amount of sales increases*. For example, the rate of commission may be 8% on all sales up to Rs10,000 and 10% on all sales over ₹10,000. In many cases a salesman receives a fixed salary plus a commission.

Illustration 3.12 A salesman who works on a commission basis receives 12% of his sales. How much was his commission on a sale amounting to ₹14,575?

Solution:

Commission received = 12%

His total sales = ₹4,575

Then, Commission = 12% of ₹14,575

$= \frac{12}{100} \times 14,575$

= ₹1,749

So, commission received by salesman = ₹**1,749.**

Illustration 3.13 A sales clerk receives a weekly salary of ₹6,600 plus a commission of 5% on all sales above ₹27,500 per week. During three weeks her total sales were ₹84,700, ₹67,925 and ₹57,200. What were her total earnings for the three weeks?

Solution:

Weekly salary = ₹6,600

Commission received on sales above ₹27,500 = 5%

Salary for three weeks = ₹6,600 x 3 = ₹19,800

Sales exceeding ₹27,500:- 1st week = ₹84,700 – ₹27,500 = ₹57,200

2nd week = ₹67,925 – ₹27,500 = ₹40,425

3rd week = ₹57,200 – ₹27,500 = ₹29,700

Total Sales exceeding ₹27,500 = ₹57,200 + ₹40,425 + ₹29,700 = ₹1,27,325

Commission = 5% of ₹1,27,325

= 5 /100 x 1,27,325 = ₹6,366

Total earnings = ₹19,800 + ₹6,366 = ₹**26,166.**

Illustration 3.14 A salesclerk is paid an 8% commission up to ₹Rs 44,000 of weekly sales and a 14% commission on all sales over ₹44,000. Last week his sales were ₹71,500. What was his commission?

Solution:

Commission received upto ₹44,000 = 8%

Commission received over ₹44,000 = 14%

Sales above ₹44,000 = ₹71,500 – ₹44,000 = ₹27,500

Commission = 8% of ₹44,000 + 14% of ₹27,500

= 3,520 + 3,850

= ₹7,370

Student Activity

8. A real estate agent receives a commission of 6% of the selling price of a house. If he sells a house for ₹,35,000 what is his commission? What are the net proceeds?
9. A salesman receives a weekly salary of Rs 8,800 plus a commission of 4% on all sales above ₹99,000. If his sales for four successive weeks were ₹1,19,900, ₹1,07,250, ₹1,35,960, and Rs 1,21,000, find his total earnings from salary and commission for the four weeks.

3.4 RATIOS

A ratio is a comparison of two quantities by division. Since ratio is an abstract number, the two quantities that are being compared must be expressed in the same unit. Ratio represents the relation that one quantity bears to the other. It indicates what multiple or part one quantity is of the other. In other words, *ratio of two quantities represents the number of times one quantity contains another quantity of the same kind.* **For example**, supposing one has 8 oranges and 6 lemons in a bowl of fruit, the ratio of oranges to lemons would be 4:3 (which is equivalent to 8:6) while the ratio of lemons to oranges would be 3:4. Additionally, the ratio of oranges to the total amount of fruit is 4:7 (equivalent to 8:14). The 4:7 ratio can be further converted to a fraction of $\frac{4}{7}$ to represent how much of the fruit is oranges.

The quantities being compared in a ratio might be physical quantities such as speed or length, or numbers of objects, or amounts of particular substances. A common example of the last case is the weight ratio of water to cement used in concrete, which is commonly stated as 1:4. This means that the weight of cement used is four times the weight of water used. It does not say anything about the total amounts of cement and water used, nor the amount of concrete being made. Equivalently it could be said that the ratio of cement to water is 4:1, that there is 4 times as much cement as water, or that there is a quarter (1/4) as much water as cement..

The Basic Steps for Solving Problems

The most common ratio problems involve a comparison between two quantities. These ratios are called ***two-term ratios***. There are three basic steps you must do first, when working any ratio problems:

1. *Change the quantities to the same units; then reduce the ratio to its simplest form*.

   For example, what is the ratio of 6 minutes to 8 hours?

   First, change the hours to minutes:

   8 hours = 8 × 60 = 480 minutes

Write the ratio as a fraction and simplify:

$$\frac{6}{480} = \frac{1}{80}$$

We found that the ratio of 6 minutes to 8 hours is 1:80.

2. Write the items in the ratio in fraction form.
3. Make sure that there are the same items in the numerator and denominator.

For example, if the ratio of Olga's classical CDs to her rock CDs is 14 to 25, the right setup is this:

$$\frac{\text{classical}}{\text{rock}} = \frac{14}{25}$$

Illustration 3.15 In a bag of blue and yellow candies, the ratio of blue candies to yellow candies is 3:5. If the bag contains 60 yellow candies, how many blue candies are there?

Solution:

Let the number of blue candies be x Next, write the items in the ratio as a fraction:

$$\frac{blue}{yellow} = \frac{3}{5} = \frac{x}{60}$$

Solve the equation by cross-multiplication:

$3 \times 60 = 5x$

$180 = 5x$

Divide both sides by 5:

x = 36

There are 36 blue candies in the bag.

Illustration 3.16 A room is 16 foot, and 8 inches long, and the ratio of the length to the width is 4 to 5. What is the width of the room?

Solution:

Since the length is given in both feet and inches, let's convert it to inches using the fact that 1 foot equals 12 inches. To find how many inches are in 16 feet, we multiply 16 feet by 12 inches:

16 feet, and 8 inches = (16 × 12) + 8 = 192 + 8 = 200

We found that the length is 200 inches.

Let x represent the width. We can now set up the equation:

$$\frac{width}{length} = \frac{4}{5} = \frac{x}{200}$$

Solve the equation by cross-multiplication:

$$4 \times 200 = 5x$$

$$800 = 5x$$

Divide both sides by 5:

$$x = 160 \text{ inches}$$

We found that the width is 160 inches.

Let's now convert inches to feet so that the units for the width are consistent with the units for the length. Since 1 foot is 12 inches, we divide 160 inches by 12 to find out how many feet are in 160 inches:

160:12 = 13, with the remainder of 4 inches. The width is 13 feet, and 4 inches.

The room width is **13 feet, and 4 inches.**

Illustration 3.17 A school has 300 students. If the ratio of boys to girls is 31 to 44, how many more girls are there in the school?

Solution:

Since we don't know the exact number of either boys or girls, we can't set up the equation right away. We assign variables first. Let x be the number of girls; then the number of boys will be 300 − x, since there are 300 students overall.

Now we can set up the equation by writing the number of students in the ratio as a fraction:

$$\frac{boys}{girls} = \frac{31}{44} = \frac{300 - x}{x}$$

Solve the equation by cross-multiplication:

$$31x = 44(300 - x)$$

Distribute the right side:

$$31x = 13{,}200 - 44x$$

Isolate the variable and collect like terms:

$$31x + 44x = 13{,}200$$

$$75x = 13{,}200$$

Divide both sides by 75:

$$x = 176$$

We found that there are 176 girls.

In order to find the number of boys, subtract the number of girls from the number of all students:

$$300 - 176 = 124$$

We found that there are 124 boys in the school.

By subtracting the number of boys from the number of girls, we can find out how many more girls there are in the school:

$$176 - 124 = 52$$

There are **52 more girls** than boys in the school.

Illustration 3.18 At a small college, the ratio of men to women is 9:4. If there are presently 720 women, how many additional women would it take to reduce the ratio of men to women to 2:1?

Solution:

To answer the problem's question, we need to know the present number of men at the college. Let this number be x. We use the first ratio 9:4 and the present number of women to find the present number of men:

$$\frac{men}{women} = \frac{9}{4} = \frac{x}{720}$$

Solve the equation by cross-multiplying:

$$9(720) = 4x$$

$$6{,}480 = 4x$$

Divide both sides by 4:

$$x = 1{,}620$$

We found that there are presently 1,620 men at the college.

To reduce the ratio to 2:1, the college needs to take in more women. Let that number of additional women be *y*. We use the variable *y* since we already used the variable *x* to represent the amount of men. The present number of women is 720, so the new amount of women will be 720 + *y*.

Now, we know the present amount of men (this should not be changed) and we know the number of women (720 + *y*) it will take to change the ratio to the desirable ratio 2:1, so we can set up our equation:

$$\frac{men}{women} = \frac{2}{1} = \frac{1620}{720 + y}$$

Solve the equation by cross-multiplication:

$$2(720 + y) = 1{,}620$$

Distribute and isolate the variable:

$$1{,}440 + 2y = 1{,}620$$

$$2y = 1{,}620 - 1{,}440$$

Collect like terms:

$$2y = 180$$

Divide both sides by 2:

$$y = 90$$

It would take **90 additional women** to reduce the ratio of men to women to 2:1.

Student Activity

12. A completes a piece of work in 6 days. B takes 4 days to do the same work. Find the ratio of their rate of doing work.
13. A tap is used to fill tanks. Tank P gets filled in 10 hours and tank Q gets filled in 8 hours. Find the ratio of capacities of the two tanks.

Summary

- A percentage is a number or ratio expressed as a fraction of 100. It is often denoted using the per cent sign, "%".
- Sales commissions are paid to employees or companies that sell merchandise in stores or by calling on customers. The commission is meant to motivate sales persons to sell more. A commission may be paid in addition to a salary or instead of a salary. A common place where commissions are paid is in real estate marketing.
- A ratio is a relationship between two numbers of the same kind (*e.g.*, objects, persons, students, spoonfuls, units of whatever identical dimension), expressed as *"a* to *b"* or *a:b*, sometimes expressed arithmetically as a dimensionless quotient of the two that explicitly indicates how many times the first number contains the second.

Glossary

Percentage: The per cent means per hundred or out of hundred. It expreses one quantity as a percentage of another quantity.

Commission: The payment of commission as remuneration for services rendered or products sold is a common way to reward sales people. Payments often will be calculated on the basis of a percentage of the goods sold.

Ratios: A ratio is a comparison of the sizes of two or more quantities of the same kind by division.

Answers to Self Assessment Questions

1. $16\frac{2}{3}\%$
2. ₹ 3,00,000
3. N = 600

9. ₹38,724.40
10. ₹22,880
11. ₹3,129.50

4. 80%
5. ₹ 285
6. ₹ 4,000
7. ₹ 72.576 lakhs
8. ₹ 8,100; ₹1,43,100
12. 2:3
13. 5:4
14. 7:6
15. ₹10,500

Review Questions

1. Vipul decided to donate 5% of his salary. On the day of donation he changed his mind and donated ₹1687.50 which was 75% of what he had decided earlier. How much is Vipul's salary?
2. In a library, 40% of the books are in English, 40% of the remaining books are in Hindi. Remaining books are in Oriya. If there are 4800 books in Hindi, then what is the total number of books in the library?
3. Three-fifth of a number is 30 more than 50 per cent of that number. What is 80 per cent of that number?
4. If x is 90% of y, what per cent of x is y?
5. Mr. Yadav spends 80% of his monthly salary on consumable items and 50% of the remaining on clothes and transport. He saves the remaining amount. If his savings at the end of the year is Rs 5730, how much amount per month he would spent on clothes and transport?
6. Ashok gave 40% of the amount he had to Jayant, in turn gave one-fourth of what he received from Ashok to Prakash. After paying ₹200 to the taxi driver out of the amount he got from Jayant, Prakash now has Rs 600 left with him. How much amount did Ashok have?
7. A box contains 20 paise, 10 paise and 5 paise coins in the ratio 5 : 3 : 4. If the total amount in the box is ₹15, find the number of coins of each type.
8. A mixture contains milk and water in the ratio 5:2. If 14 litres of water is added to the mixture, the ratio changes to 2:5. Find the quantity of milk in the original mixture.
9. ₹ 1,350 is distributed among 2 men, 3 women and 5 children so that the shares of a man, a woman and a child have the ratio 10:5:2. Find the share of a man.
10. A father divides 780 kg of rice between his two sons such that the elder gets twice of his younger brother. Find the share of each.

Further Readings

A.K. Arte & R.V. Prabhakar, *A textbook of Mathematics*

R.S. Bhardwaj, *Business Mathematics*, Excel Books.

UNIT 4 DISCOUNTING AND FACTORING TECHNIQUES

Unit structure

- Introduction
- Discount
- Factoring
- Mark-up pricing
- Summary
- Keywords
- Review Questions
- Further Readings

Learning Objectives

After reading this unit you should be able to:

- Meaning - Discount
- Basic concepts
- Factoring Techniques
- Selling price of an items; given its cost and rate of mark up.
- Cost of an item; given its selling price and rate of mark up.
- Mark up rate, given the cost and selling price of an item.

4.1 INTRODUCTION

A good way to save money is to shop when merchandise is on sale. After Christmas many stores reduce the prices of toys, furniture, and other household items. In late February and March winter clothing usually is cleared out at lowered prices. Following the fourth of July there are reductions on summer items.

The amount that an article is reduced in price is called a **discount**. The rate of discount is the rate of per cent that is taken off the original price of the article. *The original price of an article is known as the* **list price or marked price**, while *the amount for which the article sells after the discoun*t has been subtracted is the **net price or sale price**.

4.2 MEANING – DISCOUNT

Discount is defined as the reduction made from the amount of bill instead of its immediate cash payment or bulk purchase by a trader. In practical terms; *discount is defined as the difference between present value and the total amount after a certain period of time*. **For example**, a dealer allows a discount to customer who makes the payment in cash and he allows a 5% discount on the bill of₹200. Then, he will charge ₹190 instead of ₹s200. This reduction is taken as discount.

4.3 BASIC CONCEPTS

a) **True discount**: *True discount refers to the difference between amount due and its present value or present worth*. It corresponds to interest. It is denoted by T.D.

b) **Present value or present worth**: *Present value or present worth is defined as the sum due at the end of a given period*. It corresponds to the principal. It is denoted by p.w.

c) **Amount**: *The sum of discount and present worth is known as the* **amount due (A)**. The relation between true discount, present value or worth and amount is:

True Discount (T.D.) = Interest on present worth
Amount = Present worth value + Discount

d) **Banker's Discount**: *When the money is withdrawn from the bank before its due date, the bank makes the same deduction from the face value of the bill*. This is called the **banker's discount**. In other words, banker's discount is the simple interest on the amount of bill for the number of days the bill has yet to run. It is denoted by B.D.

Banker's Discount (B.D.) = Interest on bill value
Banker's Gain (B.G.) = Banker's Discount – True Discount
B.G. = B.D. – T.D.

In banker's discount, the computation of period is a prime factor. It is calculated from the date of discounting the bill to the legally due date, and the number of days calculated should be divided by 365 if the rate is per annum.

Note:

1. The term 'months' in the bill always stands for calendar months. The year is supposed to contain 365 days.
2. Three days of grace is added only when the actual dates are given. If the bill is payable on demand or no date is given, only the period of time is stated, three grace days are not allowed.

e) **Bills of exchange**: When a trader buys goods from a wholesaler, the payment is not made in cash but through bills of exchange. It is a written document to make payment for goods after a certain time. There are three parties involved in a bill of exchange: drawer, drawee and payee. The drawer writes the bill, drawee is directed to pay the bill. The third party is the payee to whom the money is to be paid. It may also be called *hundi* in commercial circles.

Note:

1. The bill of exchange is always drawn on standard paper.
2. A date on which a bill is payable is called the due date. A bill drawn at 6 months is nominally due after 6 calendar months, not after 180 days.
3. If a bill of exchange is not payable on demand, it will mature on the third day after the normal due date. These three days are known as *days of grace*. Suppose, a bill is drawn on 20 November 2011 at 6 months. It will nominally be due on 20th May 2012, but legally it will be due on 23rd May 2012.

4.4 FORMULAS FOR DISCOUNTING TECHNIQUES

1. **Present Worth (P.W)** = $\dfrac{\text{Sum X 100}}{(100+\text{R X T})}$

2. **True Discount (T.D.)** = $\frac{\text{Sum X R X T}}{(100\text{+R X T})}$

3. **Banker's Discount (B.D.)** = B.G. + T.D.

4. **Sum due** = $\frac{\text{B.D.X T.D.}}{\text{B.D.X T.D.}}$ or, P.V. + T.D.

5. **T.D.** = $\sqrt{\text{P.W.X B.G.}}$

6. **P.V.** = B.V. – T.D.

Where, R = rate of interest and T represents the time.

Illustration 4.1 Find the present worth of ₹1150 due 3 years hence at 5 per cent per annum simple interest. Find the true discount also.

Solution:

Let the present worth = ₹100

Interest on ₹100 for 3 years at 5% = ₹$\frac{100 \ X \ 3 \ X \ 5}{100}$ = ₹15

Amount of ₹100 = ₹100 + ₹15 = ₹115

Present worth of ₹115 = ₹100

P.W. of ₹1150 = $\frac{100}{115}$ x 1150 = ₹1000.

True Discount of ₹1150 = Amount – P.W. = ₹1,150 – ₹1,000 = **₹150**.

Illustration 4.2 A offers ₹4000 for a T.V. and B offers ₹4,444 to be paid in 2 years. Which is better offer and by how much money is being reckoned at 5% simple interest?

Solution:

In order to compare the two offers, we should find the P.W. of ₹4,444

Now Sum = ₹4,444, Rate = 5% T = 2 years

$$\text{P.W.} = \frac{4444 \times 100}{(100+2 \times 5)} = \frac{4444 \times 100}{110} = ₹4040$$

Now first offer = ₹4000 and second offer = ₹4040

Hence, the second offer is better by ₹**40**.

Illustration 4.3 If the discount on ₹500 due 2 years hence is ₹100, find the rate of interest.

Solution:

Given discount is ₹100, Sum = ₹500 and Time = 2 years

P.W. = ₹500 – ₹100 = ₹400

Discount on amount on interest on P.W. = ₹100, Time = 2 years

∴ ₹100 is S.I. on 400 for 2 years

Hence, Rate = $\frac{100 \times 100}{400 \times 2}$ = $\mathbf{12\frac{1}{2}\%}$

Illustration 4.4 If ₹10 be allowed as discount on a bill of 90 for one year, how much should be allowed on a bill of the same amount due two years hence?

Solution:

₹10 is the discount on ₹90 for one year

S.I. on ₹ (90 – 10), that is, ₹80 for one year = ₹10

So, S.I. on ₹80 for 2 years = ₹20

Amount after 2 years = ₹80 + ₹20 = ₹100

Now discount on ₹100 = ₹20

Discount on ₹90 = ₹$\frac{20}{100}$ x 90 **= ₹18.**

Illustration 4.5 Find the difference between simple interest and true discount on ₹575 in 3 years at 5 per cent per annum.

Solution:

Sum = ₹575, T = 3 years, r% = 5%

S.I. = $\frac{Sum\ X\ R\ X\ T}{100}$ = ₹$\frac{575\ X\ 3\ X\ 5}{100}$ = ₹$\frac{8,625}{100}$ = ₹86.25

Discount = $\frac{Sum\ \ X\ R\ \ X\ T}{100+(R\ X\ T)}$ = ₹$\frac{575\ X\ 3\ X\ 5}{115}$ = ₹75

Difference between interest and discount = ₹ (86.25 – 75) = **₹11.25**

Illustration 4.6 If the interest on ₹6400 be equal to true discount on ₹6560 at 5%, when is the latter sum due?

Solution:

S.I. on ₹6,400 = T.D. ₹6,560

Rate = 5%

Evidently amount = ₹6560, P.W. = ₹6400

Discount =₹6560 – ₹6400 = ₹160

Time = $\frac{160\ X\ 100}{5\ X\ 640}$ **= $\frac{1}{2}$ year or 6 months.**

Illustration 4.7 A bill for ₹2000 drawn on 20th December 2010 at 6 months is discounted on 11th April 2011. If the payment made by the banker is ₹1978, find the rate of interest. Find also rate the banker got on his money.

Solution:

Bill drawn on 20th Dec. 2010 at 6 months

∴ legally due date = 23rd June 2011

Discounted on 11th April 2011

So, total number of days = 19 days of April + 31 days of May and 23 days of June = 73 or $\frac{1}{5}$ year.

i. Now, the amount of the bill = ₹2000
and the banker payment = ₹1978
∴ B.D. = ₹ (2000 – 1978) = ₹22

That is, ₹22 is the interest on ₹2000 for $\frac{1}{5}$ year

∴ Rate of interest = $\frac{22\ X\ 100}{2000\ X\ \frac{1}{5}}$ = $\frac{11}{2}$ = **5 $\frac{1}{2}$ %**

ii. Now, Banker paid ₹1978 only, that is, he got ₹22 (as interest) on ₹1978 in $\frac{1}{5}$ year.

∴ the rate which the banker got = $\frac{22 \times 100}{1978 \times \frac{1}{5}}$ = **5 $\frac{555}{989}$ %.**

Illustration 4.8 Find the difference between true and banker's discount on ₹975, due 9 months hence at 5 $\frac{1}{3}$ % per annum.

Solution:

Sum = ₹975, Time = $\frac{9}{12}$ or $\frac{3}{4}$ year, R = $\frac{16}{3}$ %

Using existing formula to compute B.D.

B.D. = $\frac{Sum \times R \times T}{100}$

T.D. = $\frac{Sum \times R \times T}{100+(R \times T)}$

i. ∴ B.D. = ₹$\frac{975 \times 16 \times 9}{100 \times 3 \times 12}$ = ₹39

ii. T.D. = ₹$\frac{975 \times \frac{16}{3} \times \frac{3}{4}}{100+\frac{16}{3} \times \frac{3}{4}}$ = $\frac{975 \times 4}{104}$ = ₹$\frac{75}{2}$ = ₹37.50

∴ The difference between B.D. and T.D. = ₹39 – ₹37.50 = **₹1.50.**

Illustration 4.9 The banker's discount on ₹ 1208 at 5% is the same as true discount on ₹ 1223.10 for the same time and the same rate. Find the time.

Solution:

B.D on ₹1208 = T.D. on ₹1223.10

∴ P.W. is given by ₹ 1208

Then interest on ₹1,208 at 5% per annum = ₹ (1,223.10 – 1,208) = ₹15.10

∴ Time = $\frac{I \times 100}{Sum \times R}$

Time = $\frac{\frac{151}{10} \times 100}{1208 \times 5}$ yr. = $\frac{1}{4}$ yr = **3 months.**

Illustration 4.10 A bill is payable after 6 months hence. If its true discount at 5% per annum be ₹ 73.25. Find its present value and total amount.

Solution:

Given T.D = ₹73.25

S.I. on P.W = ₹73.25

$$\text{P.W} = \frac{S.I \; X\; 100}{r \; X \; t} = ₹\,\frac{73.25 \; X \; 100}{5 \; X \frac{1}{2}} = ₹\,\frac{7325}{5} X\, 2 = ₹2930$$

∴ Amount = P.W. + T.D = ₹ (2930 + 73.25) = ₹ **3003.25**

Illustration 4.11 A money lender discounted a bill due 10 months hence at 4%, what rate of interest did he get on his money?

Solution:

Time = 10 months = $\frac{10}{12}$; rate = 4%

Let the face value of the bill be ₹100, then

$$\text{B.D.} = \frac{Sum \; X \; R \; X \; T}{100} = \frac{100 \; X \; 4 \; X \; \frac{10}{12}}{100} = ₹\,\frac{10}{3}$$

The banker actually invests ₹ (100 - $\frac{10}{3}$)

That is, ₹= $\frac{290}{3}$ to get Rs 100 after 10 months, so he will get an interest of ₹$\frac{10}{3}$ on a sum of ₹$\frac{290}{3}$ after 10 months.

$$\text{Rate of interest, he gets} = \text{B.D.} = \frac{I \; X \; 100}{T \; X \; Sum} = \frac{\frac{10}{3} \; X \; 100}{\frac{290}{3} \; X \; \frac{10}{12}} \% = \frac{100 \; X \; 12}{290} \% = \mathbf{4\frac{4}{29}} \; \%$$

Illustration 4.12 True discount on a certain sum of money is ₹30 and banker's discount is ₹3 more. Find the sum.

Solution:

Here T.D. = ₹30

and B.D. = ₹ (30 + 3) = ₹33

B.G. = ₹ (33 – 30) = ₹3 = S.I. on T.D.

i.e., ₹3 is S.I. on Rs 30

We are required to find the sum on which B.D. i.e. S.I. is ₹33.

Let the sum be ₹x, then S.I. on Rs x is ₹33

$$\frac{x}{30} = \frac{33}{3}$$

(S.I. in a given period at a given rate in proportional to the principal)

$$x = \frac{30}{3} \text{ X } 33 = 330; \text{ Hence, the sum = } ₹\mathbf{330}$$

Illustration 4.13 If the true discount on a bill of ₹90 for one year is ₹10. What will be the true discount on the same sum for 2 years.

Solution:

B.V. = ₹90

T.D. = ₹10

P.V. = B.V. – T.D. = ₹ (90 – 10) = ₹80 [Time = 1year]

T.D. = Interest on P.V.

Interest on ₹80 for one year = ₹10

Interest on ₹80 for two years = ₹20

B.V. = ₹ (80 + 20) = ₹100

Since, true discount on ₹100 for two years = ₹20

True discount on ₹1 for two years = ₹$\frac{20}{100}$

True discount on ₹90 for two years = ₹$\frac{20}{100}$ x 90 **= ₹18.**

Illustration 4.14 A bill of ₹4000 drawn on a certain data was due 6 months later. It was encashed at a bank for ₹3960 on 25th March 2013. If the rate of interest was 5% per annum, on what date was it drawn?

Solution:

A bill drawn on certain date = ₹4,000 due six months

Bill was encashed for ₹3,960

Rate = 5% p.a.

B.D = ₹ [4000 – 3960] = ₹40

But B.D = Interest on B.V. for the unexpired period, say, *n* days,

$\therefore 40 = \frac{4000 X \frac{n}{365} X 5}{100} \rightarrow n = 73.$

$\therefore$ Counting 73 days from 25th march, 2013, we get

6 days for March + 30 days for April + 31 days for May + 6 days for June = 73 days

$\therefore$ The bill was legally on 6th June.

(Including the day of 25th March and excluding the day of 6th June)

Subtracting three days of grace, the bill was nominally due on 3rd June, 2013. Counting 6 months back, he bill was drawn on **3rd December, 2012**.

Illustration 4.15 Present value of a bill due 4 years hence is ₹1150. If the bill were due at the end of 2 $\frac{1}{2}$ years, its present value would have been ₹1240. Find the rate of simple interest and the face value of the bill.

Solution:s

Suppose Rate % = r

In the first case: P.W. = ₹1150; Time = 4 Years and Rate % = *r*

$$\text{Interest} = \frac{1150 X r X 4}{100} = 46r$$

$$\text{Sum Due} = (1150 + 46r) \qquad ...(1)$$

In the second case: P.W. =₹1240; Time = 2 $\frac{1}{2}$ years and Rate % = *r*

$$\text{Interest} = \frac{1240 X 5}{100 X 2} X r = 31r$$

$$\text{Sum Due} = (1240 + 31r) \qquad ...(2)$$

From (1) and (2), we get, $1150 + 46r = 1240 + 31r$

$$46r - 31r = 1240 - 1350$$

$$15r = 90$$

$$r = 6\%$$

From (1) Sum Due = ₹ (1150 + 46r) = ₹ (1150 + 46 X 6)

= ₹ (1150 + 276) = ₹**1426**

Illustration 4.16 A banker discounts a bill for a certain amount which has 73 days to run before it matures at 5%. The discounted value of the bill is ₹995.62. Find the face value of the bill.

Solution: Let face value of the bill = ₹100

Number of days for which bill has yet to run = 73 days = $\frac{73}{365}$ year = $\frac{1}{5}$ years

Rate = 5%

B.D. = $\frac{Sum\ X\ R\ X\ T}{100}$

B.D. = $\frac{100\ X\ 5\ X\ 1}{100\ X\ 5}$ = Re 1

Discounted value of the bill = (100 – 1) = ₹99

If discounted is ₹.99, face value = ₹100

If discounted is ₹1, face value = ₹$\frac{100}{99}$

If discounted is ₹995.62 face value = ₹$\frac{100}{99}$ X 995.62 = ₹**1005.70**(approx)

4.5 FACTORING TECHNIQUES

Factoring techniques are defined as those techniques which segregate the total into very small parts. They are useful tools in modern business world because they help in dividing the share in different factors of production. They are also helpful in increasing or decreasing the shares proportionally as required by business enterprises.

Factoring techniques enable the businessmen infusing the business problems and encounter them. They help businessmen from the beginning to the end, that is, from purchase of raw material to monopolization of the product in the market.

Student Activity

1. A man bought a cycle for ₹300 and sold it for ₹440 at a credit of one year. What is the gain per cent reckoning money worth 10 per cent per annum?
2. The difference between the interest and the true discount on a certain amount of money for 6 months at 4 per cent is ₹20. What is sum?
3. A bill drawn for ₹5,050 on 20 June at five months. It is discounted on 11 September at 5 per cent per annum. How much does the holder of the bill receive and what is the gain in the transaction?
4. A bill is drawn for ₹5,050 on 10 June, 2011 at 5 months. It is discounted on 1, Sept. 2011 at 5% per annum. How much does the holder of the bill receive and what is the gain of the banker?
5. If the difference between the true discount and banker's discount on a sum due in 4 months at 3% is Rs 20, find the face value of the bill.

4.6 MARK UP PRICING

Stores buy items from a wholesaler or distributer and increase the price when they sell the items to consumers. The increase in price provides money for the operation of the store and the salaries of people who work in the store.

A store may have a rule that the price of a certain type of item needs to be increased by a certain percentage to determine how much to sell it for. This percentage is called the **markup**.

If the cost is known and the percentage markup is known, the sale price is the original cost plus the amount of markup. Sale Price = Cost + Mark Up. **For example**, if the original cost is ₹60.00 and the markup is 25%, the sales price should be ₹60.00 + ₹60.00 X $\frac{25}{100}$ = ₹75.00.

A faster way to calculate the sale price is to make the original cost equal to 100%. The markup is 25% so the sales price is 125% of the original cost.

In the example, ₹60.00 X $\frac{125}{100}$ = ₹75.00

MARKUP FORMULAS:

Selling Price = Cost + Mark up

Mark up = Mark up rate X Cost

Illustration 4.17 A Jewelry Store uses a mark up rate of 70% on all items. If a pair of ear rings cost the store ₹550.00, find the selling price.

Solution:

Cost of a Pair of ear rings = ₹550

Mark up = Mark up Rate x Cost

Mark up = (.70) (₹550.00)

Mark up **= ₹385.00**

Selling Price = Cost + Mark up

Selling Price = ₹550.00 + ₹385.00

Selling Price = **₹935.00**

Illustration 4.18 A television sells for ₹32,000. The markup is 60% of the cost. Find the cost of the television.

Solution:

Selling Price of T.V. = ₹32,000

Mark Up rate = 60%

Selling Price = cost + mark up

₹32,000 = cost + mark up

₹32,000 = cost + mark up rate x cost

₹32,000 = cost +.60(cost)

(*Note: the selling price is the cost of the TV plus 60% of that cost or 160% of the cost or 1.60* x *cost)*

₹32,000 = 1.60 x cost

₹32,000 ÷1.60 = cost

₹20,000 = cost

Thus, cost of television = **₹20,000**

Illustration 4.19 The cost of a baseball bat is ₹660.00, the Sporting Goods Store sells it for ₹ 990.00. Find the markup rate.

Solution:

Cost of a baseball bat = ₹660

Selling Price is ₹990

Selling Price = Cost + Mark up

₹990 = ₹660 + Mark up

₹330 = Mark up

Markup = Mark up rate x Cost

₹330 = Mark up rate x ₹660

Markup rate = $\frac{330}{}$ = $\frac{1}{}$ = 0.5 **= 50%**

Summary

- Discounts and allowances are reductions to a basic price of goods or services. They can occur anywhere in the distribution channel, modifying either the manufacturer's list price (determined by the manufacturer and often printed on the package), the retail price (set by the retailer and often attached to the product with a sticker), or the list price (which is quoted to a potential buyer, usually in written form).
- Certain securities are available for purchase by retail investors from dealers who sell the securities directly from their own accounts. The dealer's only compensation for the sale comes in the form of the mark up, the difference between the price (the security was purchased at) and the price (the dealer charges to the retail investor).

Glossary

Discount: The condition of the price of a bond that is lower than par. The discount equals the difference between the price paid for a security and the security's par value.

Mark up: Mark up refers to the value that a player adds to the cost price of a product. The value added is called the mark-up. The mark-up added to the cost price usually equals retail price

Answers to Self Assessment Questions

1. $33\frac{1}{3}\%$
2. ₹51,000
3. 50p
4. ₹0.50
5. ₹20,200

Review Questions

1. True discount on a certain sum of money due six months hence is ₹24 and the interest on the same for the same time and at the same rate is ₹25. Find the sum and the rate per cent.
   (Ans: ₹600)

2. A tradesman's terms are 20% discount for each payment and interest is charged after 6 months. What rate of interest per annum does the customer get on his money for cash payment?
   (Ans: 50%)

3. The Banker's Discount and True Discount on a certain sum of money at 5% simple interest and respectively ₹30 and ₹25 for the same period. Find the sum and the period. (Ans: 4 years)

4. The true discount of ₹151.20 at 12% per annum is ₹7.25. What sum due to the same period hence will have ₹50 as true discount, the rate of interest being 15% per annum? (Ans: ₹850)

5. A bill for ₹10,100 was drawn on 12th June 2012 for 5 months. It was discounted on 3rd Sept 2012 at 5% p.a. find (*a*) B.D. (*b*) B.G. (*c*) How much does the holder of the bill receive?

6. A dress was on sale for ₹7,500. Find the original price if the dress had been discounted 25%.

7. A computer company uses a mark up rate of 50%. If a computer game costs the company ₹40,000, find the selling price.

8. A tennis racket is sold for ₹2,500. If the cost to the store was ₹2,000, find the mark up rate.

9. A set of golf clubs sells for ₹30,000. If the clubs were marked up 20%, find the cost of the clubs.

Further Readings

P.N. Arora, *Mathematics*, S. Chand

R.S. Bhardwaj, *Business Mathematics*, Excel Books.

UNIT 5 CORPORATE STOCKS AND BOND'S APPLICATIONS

Unit structure

- Introduction
- Stock buy and sell
- Gain and loss on stock
- Rates of yield
- Summary
- Keywords
- Review Questions
- Further Readings

Learning Objectives

After reading this unit you should be able to:

- Compute costs and proceeds of stocks
- Return on bonds investment
- Know Yield measures

5.1 INTRODUCTION

The investor takes a number of decisions in the process of investment. The investor has to decide about his risk tolerance level and the nature of assets to be bought; whether they are stocks or bonds or real estates. Once he decides the nature of the assets, he has to select it from the different alternatives. *For example*, if the common stock is chosen by the investor he has to decide which company's stock he has to buy. It may be the Reliance Industry stock, or the BHEL or Infosys or any other company's stock. Stocks are selected on the basis of costs and proceeds.

5.1.1 Computing the costs and proceeds of stock transactions

Cost basis *is the price paid to acquire shares plus commissions and any fees.* Stock plans enable employers to issue company stock for services rendered. As a result of this exchange, ordinary income (compensation) may be realized as part of the equity transaction. The event that triggers the ordinary income varies and is dictated by tax law, but can include grant, vest, or exercise of the award, and the subsequent sale or disposition of those shares. Ordinary income is a factor in determining cost basis when stock plan shares are sold.

Cost basis is used to compute capital gains and losses. You will need to determine the cost basis of a sold security in order to accurately file your taxes.

The example here is for illustrative purposes only, using the following assumptions:

• 100 shares are acquired and subsequently sold.

• In Scenario *A*, the shares are purchased on the stock market.

• In Scenario *B*, the shares are acquired via an option exercise. The option exercised is a non-qualified stock option issued pursuant to a company stock plan.

Sale Proceeds	Adjusted Cost Basis	Capital Gain or Loss
Proceeds from selling an investment after deducting costs like commissions and fees.	The original purchase price for an investment plus any fees, commissions, or other adjustments.	Gains and losses must be reported at tax time.

Example A: Open Market Purchase

Shares sold	100	Shares bought	100	
Price per share	X 20	Price per share	X 10	
Gross Proceeds	2,000	Cost basis	1,000	
Commissions	-20	Commissions	+20	
Sale proceeds	1,980	Sale Proceeds	1,020	Net = 960

Example B: Option exercise

Shares sold	100	Shares bought	100	
Price per share	X 20	Price per share	X 10	
Gross Proceeds	2,000	Exercise (purchase) price	1,000	
Commissions	-20	Ordinary income	950	
Sale proceeds	1,980	Sale Proceeds	1,950	Net = 30

After purchasing stock, a buyer may sell the stock at any price on the open market, regardless of the par value. Stocks are usually bought and sold on stock exchanges, the formal market places set up for the purpose of trading stock. Major exchanges in India are Bombay Stock Exchange, National Stock Exchange, A stock broker usually handles stock transactions the purchase and sale of stocks for clients. Today many people also trade via internet.

Both the buyer and the seller of stock pay commission to the stock broker. *The total amount paid by a buyer to purchase a stock includes the market price of the stock and the stock broker's commission (charge)*. The proceeds received by the seller are equal to the selling price minus the commission.

Total amount paid by buyer = Market price of stock + Commission

Proceeds received by seller = Selling price – Commission

Broker's commission may be flat rate per transaction a per cent of the value of the stock, an amount per share traded, or an amount negotiated between the client and the broker.

Calculation of cost of purchasing stock

STEP 1: Calculate the Cost of the shares

Cost of shares = Price per share x Number of shares

STEP 2: Compute the amount of the broker's commission

Broker's Commission = Cost of shares x Commission rate

STEP 3: Determine the total cost of the stock purchase

Total Cost = Cost of shares + Broker's Commission

Calculation of proceeds from selling stock

STEP 1: Calculate the value of shares on sale

Value of shares = Price per share x Number of shares

STEP 2: Compute the amount of Broker's Commission

Broker's Commission = Cost of shares x Commission rate

STEP 3: Determine the proceeds by subtracting the commission from the value of the shares

Proceeds = Value of shares – Broker's commission

5.1.2 Computing the costs and proceeds of round and odd lots

Round Lots

A group of 100 shares of a stock, or any group of shares that can be evenly divided by 100, such as 500, 2,600 or 14,300. A round lot has historically been the smallest order that can be placed through an exchange. However, "round-lot one" now allows for the execution of orders as small as one share on some exchanges. A round lot may also be referred to as a "normal trading unit".

Odd Lots

An order amount for a security that is less than the normal unit of trading for that particular asset. Odd lots are considered to be anything less than the standard 100 shares for stocks. Trading commissions for odd lots are generally higher on a percentage basis than those for standard lots, since most brokerage firms have a fixed minimum commission level for undertaking such transactions.

Stocks are sold in round lots, odd lots or a combination of the two. A round lot usually is 100 shares. An odd lot consists of any number of shares less than 100 (1 to 99 shares) is an odd lot for a stock with a 100-share round lot. When odd lots are purchased a small extra charge or odd lot differential is commonly added to the round lot price. The differential is added to the price for a purchaser and deducted from the price for the seller. We use a differential of 0.125 paisa per share as the odd lot rate.

Another feature that sometimes makes preferred stock an attractive investment is the possibility of converting the preferred stock into common stock. Convertible preferred stock gives the owner the option of converting those preferred shares into a stated number of common shares. For example, a stated conversion of 1 to 3 means that 1 share of preferred stock could be changed into 3 shares of common stock. The conversion feature combines the safety of preferred stock with the possibility of growth through conversion to common stock.

Illustration 5.1: Ajay purchase 350 shares of Mercury manufacturing common stock at ₹46.50 per share. A few months later he sells the shares at ₹54.31. Stock broker charges 3% commission on round lots and 4% on odd lots. Calculate a) Total cost b) Proceeds c) Gain/Loss on the transaction.

Solution:

Total share purchased by Ajay = 350

Cost of each share = ₹46.50

Broker's Commission on round lots = 3%

Broker's Commission on odd lots = 4%

a) Calculation of Total Cost

STEP 1: Cost of shares = Price per share x Number of shares

= ₹46.50 x 350

= ₹16,275

STEP 2: Broker's Commission = Cost of shares x Commission rate

Round lot broker's commission = 300 x 46.50 x 0.03 = ₹418.50 [3% = 0.03]

Odd lot broker's commission = 50 x 46.50 x 0.04 = ₹93 [4% = 0.04]

Broker's Commission = ₹418.50 + ₹93 = ₹511.50

STEP 3: Total Cost = Cost of shares + Broker's Cosmmission

= ₹16,275 + ₹511.50

= **₹16,786.50**

b) Calculation of Proceeds

STEP 1: Value of shares = Price per share x Number of shares

= 54.31 x 350

= ₹19,008.50

STEP 2: Broker's Commission = Cost of shares x Commission rate

Round lot broker's commission = 300 x 54.31 x 0.03 = ₹488.79

Odd lot broker's commission = 50 x 54.31 x 0.04 = ₹108.62

Total Broker's Commission = 488.79 + 108.62 = ₹597.41

STEP 3: Proceeds = Value of shares – Broker's commission

= ₹19,008.50 - ₹597.41 = ₹18,411.09

c) Calculation of gain/loss

Gain / Loss = Value of share – Cost of share

= ₹18,411.09 – ₹16,786.50

= ₹**1,624.59**

Illustration 5.2: Karan bought 200 shares of Home depot stock at ₹36 per share. What was his cost including commission of ₹0.20 per share?

Solution:

Total shares bought by Karan = 200

Price per share = ₹36; rate of commission = 20% = 0.20

Calculation of Total Cost

STEP 1: Cost of shares = Price per share x Number of shares

= ₹36 x 200

= ₹7,200

STEP 2: Broker's Commission = Number of shares x Commission rate

= 7,200 x ₹0.20

=₹40

STEP 3: Total Cost = ₹7,200 + ₹40 = ₹**7,240**

Illustration 5.3: Karan sold 800 shares of Ashok Leyland at ₹36.17 per share, less commission of ₹0.20 per share. What were the proceeds of the sale?

Solution:

Total shares sold by Karan = 800

Price of 1 share = ₹36.17

Commission rate = 20% = 0.20

Calculation of Proceeds

STEP 1: Value of shares = Price per share x Number of shares

= ₹36.17 x 800

= ₹28,936

STEP 2: Broker's Commission = Number of shares x Commission rate

= 800 x ₹0.20

= ₹160

STEP 3: Proceeds = ₹28,936 - ₹160 = **₹28,776**

Illustration 5.4: Johny bought 500 shares of PepsiCo stock at ₹68.30. What was his cost including a flat fee of ₹19.95?

Solution:

Total shares bought by Johny = 500

Price of each share = ₹68.3

Broker's Commission = ₹9.95

Calculation of Total Cost

STEP 1: Cost of shares = Price per share x Number of shares

= ₹68.30 x 500

= ₹34,150

STEP 2: Broker's Commission = ₹19.95

STEP 3: Total Cost = ₹34,150 + ₹19.95 = **₹34,169.95**

Illustration 5.5: David purchased 300 shares of IFCI at ₹31.02 per share. He later sold the stock at ₹33.10 per share. What was his gain/loss on the purchase and sale after counting commission of ₹0.20 per share on both the purchase and the sale?

Solution:

a) **Calculation of Total Cost**

STEP 1: Cost of shares = Price per share x Number of shares

= ₹31.02 x 300

= ₹9,306

STEP 2: Broker's Commission = Number of shares x Commission rate

= 300 x ₹0.20 = ₹60

STEP 3: Total Cost = Cost of shares + Broker's Commission

= ₹9,306 + ₹60

= **₹9,36**

b) Calculation of Proceeds

STEP 1: Value of shares = Price per share x Number of shares

= ₹33.10 x 300

= ₹9,930

STEP 2: Broker's Commission = Number of shares x Commission rate

= 300 x ₹0.20 = ₹60

STEP 3: Proceeds = Value of shares – Broker's commission

= ₹ 9,930 - ₹60 = ₹9,870

c) Calculation of gain/loss

Gain / Loss = Value of share – Cost of share

= ₹9,930 – ₹9,870 = **₹504 (gain)**

Illustration 5.6: The ABC company earned ₹48,000 last year. The capital stock of the company consists of 10,000 shares of 7% preferred stock, with a par value of ₹40 per share and 50,000 shares of no-par common stock. If the board of directors declared a dividend of the entire earnings. What amount would be paid in total to the preferred and common stock and how much would each common share holder receive?

Solution:

Total number of shares = 10,000

Par value = ₹40

Total value (Preferred Stock) = Number of shares x par value

= ₹10,000 x 40

= ₹4,00,000

Payment to preferred shares = 4,00,000 x 0.07

= ₹28,000

Total Value of Common stock = Total earnings – Paid to preference shareholders

= ₹48,000 – ₹28,000

= ₹20,000

Total Value per share = $\frac{20,000}{50,000}$ = **0.40 per share**

Illustration 5.7: Assume in the above example that the preferred stock is cumulative and that for the preceding year the company had declared a dividend of only ₹16,000 or enough to pay a 4% dividend on preferred stock. The earnings of ₹48,000 for this year would be divided as follows:

Solution:

Unpaid dividend from preceding year = 7% - 4% = 3%

Cumulative dividend in arrears = 4,00,000 x 0.03 = ₹12,000

Dividend for the current year = 4,00,000 x 0.07 = ₹28,000

Total amount paid to preference stock = ₹12,000 + ₹28,000 = ₹40,000

Total earnings available for common stock = ₹48,000 – ₹40,000 = ₹8,000

Dividend per share = $\frac{8,000}{50,000}$ = **0.16** = dividend per common share

Illustration 5.8: Savita owned 200 shares of GE convertible preferred stock at ₹50 per value. She converted each share of preferred stock into 3 shares of common stock. How many shares of common stock did Savita receive when she converted?

Solution:

Number of preferred shares owned by Savita = 200

Shares of common stock = 200 x 3 = ₹600

If common stock was selling at ₹22 per share on the date of conversion, how much was Savita's common stock worth?

Value of common stock = 22 x 600 = ₹13,200

If Savita paid ₹55 per share for his preferred stock, how much had his investment increased?

Value of preferred Stock = 55 x 200 = ₹11,000

Increase in value = ₹13,200 – ₹11,000 = ₹2,200

If the converted stock pays 5% annuity and common stock usually pay ₹0.90 per share. How much more dividend might Savita expect to receive annually?

Preference stock dividend = 50 x 200 x 0.05 = **₹500**

Common stock dividend = 600 x 0.90 **= ₹540**

Extra dividend annually = ₹540 – ₹500 = **₹40.**

5.2 BOND CONTRACT

Bonds are capital market instruments and as such have maturities in excess of one year **straight bond** (plain vanilla bond, bullet bond) pays a regular (usually semi-annual) fixed coupon over a fixed period to maturity (redemption) with the return of principal (par value, nominal value) on the maturity date bonds may have more intricate flow patterns and can be classified according to many criteria.

Classification of bonds

(a) On the basis of frequency of coupon payments

- ❖ Conventional (straight, plain vanilla, bullet) bond pays regular coupons quarterly, semi-annually or annually
- ❖ Zero-coupon bond do not pay coupons at all, they are sold at a deep discount to par value and all reward from holding the bond comes in the form of capital gain
- ❖ Income bond makes coupon payment only if the income generated by the issuing firm is sufficient size of coupon payments
- ❖ Conventional bond pays a fixed coupon (determined as a percentage of its par value) over the whole life of the bond
- ❖ Variable coupon bond links its coupon to some economic variable such as the retail price index (index-linked bond), current market interest rate (floating rate note)
- ❖ Collateralised bond derives its coupons from a given package of underlying assets (ABS, asset-backed securities)

(b) On the basis of redemption date

- ❖ Conventional bond has one redemption date at maturity (five-year bond, ten-year bond, etc.)
- ❖ Double-dated bond has a range of possible redemption dates
- ❖ Callable bond has an option feature that gives the issue the right to determine the actual date of redemption
- ❖ Puttable bond has an option feature that gives the holder the right to determine the actual date of redemption

- ❖ Consol (perpetual bond, perpetuity) have no redemption date at all and interest on it will be paid indefinitely

- ❖ Convertible bond has an option feature that gives the holder the right to convert the bond at maturity into other types of bonds or into equity issuer of the bond

- ❖ Government bond is issued by a sovereign government in order to finance and manage public debt (T-Bill in India, Treasuries in USA, gilts in UK)

- ❖ Municipal bond is issue by a local authority.

- ❖ Corporate bond is issued by a private company (senior vs. junior debt, secured vs. unsecured debt, fixed vs. floating charge debt)

(c) On the basis of currency of denomination

- ❖ Domestic bond is issued by a resident issuer in the domestic currency

- ❖ Foreign bond is issued by a non-resident issuer in the domestic currency Samurai (Japan), Yankee (USA), Bulldog (UK), Matador (Spain), Kiwi (New Zealand), Alpine (Switzerland)

- ❖ Eurobond is issued and traded in a non-resident currency (other than sterling in the UK, other than euro in the Eurozone countries, etc.)

(d) Other Innovations:

- ❖ Dual currency bond: coupon payment are in one currency and the redemption proceeds are in another

- ❖ Currency change bonds: coupons are first paid in one currency and then in another

- ❖ Deferred coupon bond: there is a delay in the payment of the first coupon multiple-coupon bond: coupons change over the life of the bond in a predetermined manner

- ❖ Missing coupon bond: coupon payment is missed whenever a dividend payment on the issuer's shares is missed

- ❖ Refractable bond: attached with call and put options

- ❖ Bull or bear bond: principal on redemption depends on how a stock index has performed default or credit risk it is usually assessed in the form of credit rating (Standard & Poor's, Moody's, Fitch) the lower the credit rating, the greater the risk of default and the higher the risk premium grades of the bond: investment grades, non-investment (speculative) grades (junk bonds, high-yielding bonds)

5.3 SOURCES OF RETURNS ON BOND INVESTMENTS

The returns from investment in bonds come from the following:

1. Periodic coupon payments (except for zero coupon bonds)
2. Reinvestment income earned on the periodic coupon receipts
3. Capital gain or loss on sale of bond before maturity.

For Example: An investor who wishes to invest a certain amount of money for 5 years in the bond market has various choices available to him if the market is very liquid i.e., an active secondary market exists. The options include the following:

(a) Invest in a bond with the exact time to maturity of the investment horizon i.e., 5 years. In this case, the investor will receive periodic interest payment i.e., coupon receipts and the principal on maturity.
This investor's return is limited to sources 1 and 2 only listed above.

(b) Invest in a tenure shorter than that of the investment horizon e.g., invest in a 2-year bond and then roll over for another 2 years by investing in another 2-year bond 2 years from now. On maturity of this bond 4 years from now, the investor can then invest in a 1-year bond. All previously received cash flows i.e. reinvestment income inclusive will be invested in each instance. In this case, this investor's return will consist of items 1 and 2 only, listed above.

(c) Invest in a tenure longer than that of the investment horizon e.g., invest in a 20-year bond and sell five years at the market price for 15-year bonds of similar quality. Under this scenario, the investor will receive income from all three sources outlined above.

5.4 MEASURING YIELD

The yield on any investment is the interest rate, ***y***, that equalizes the present value of the expected cash flows from the investment to its price (cost) i.e. the interest rate, ***y***, that satisfies the equation

$$\frac{CF1}{(1+y)1} + \frac{CF2}{(1+y)2} + \ldots\ldots.+ \frac{CFn}{(1+y)n}$$

$$P = \sum \frac{CFt}{(1+y)t}$$

Where, CF_t = cash flow in period t
P = price of the investment
n = number of periods

Note: The last cash flow consists of periodic interest and the principal and
is made up of the interest **(C_i)** and Principal repayment **(P)** i.e. $\mathbf{CF_n = C_i + P}$
∴ for bonds the expression becomes:

$$CF_n = \frac{C1}{1+y} + \frac{C2}{(1+y)1} + \frac{C3}{(1+y)2} + \ldots\ldots.+ \frac{Ci}{(1+y)n} + \frac{P}{(1+y)n}$$

$$CF_n = \frac{Ci}{(1+y)n} + \frac{P}{(1+y)n}$$

The yield calculated is also called the Internal Rate of Return ***(IRR)***. Please note that the yield calculated is the yield for a period i.e., the period within which interest is paid.

Calculating the yield requires a trial and error approach (iterative process). However, for zero coupon bonds, there is no need to go through the iterative process as there are no intermediate cash flows hence determining yield is straight-forward.

$$P = \frac{CFn}{(1+y)n}$$

Solving for yield, we have:

$$y = \left(\frac{CFn}{P}\right)^{1/n}$$

Annualizing yields

As indicated above, the yield calculated is the yield per period i.e. period for which interest is paid on the bond or investment. For comparative purposes, it is important to annualize the yield. To annualize, we calculate the Effective yield as follows:

$$\textbf{Effective Yield, } y_f = (1 + \text{Periodic interest rate, } y)^m - 1$$

Where m is the number of periods for which interest is paid in a year

5.4.1 CONVENTIONAL YIELD MEASURES

Following are the measures of yield used in bond markets:
1. Nominal Yield
2. Current Yield **(CY)**
3. Yield to Maturity **(YTM)**
4. Yield to Call **(YTC)**
5. Realized (Horizon) yield

1. NOMINAL YIELD

It is the coupon rate of a particular issue. A bond with an 8% coupon has an 8% nominal yield. It ∴ provides a convenient way of describing the coupon characteristics of the bond.

2. CURRENT YIELD

Relates the annual coupon rate to market price i.e. Current Yield (CY) = Annual Rupee Coupon Interest Price This yield measure relates the total nominal values of the annual coupon cash flows to the Market price. It therefore does not take the timing of these cash flows into consideration. Also, it does not take any other source of return on the bond into consideration e.g., capital gains or losses. It is, however, an important measure to income oriented investors.

3. YIELD TO MATURITY

The Internal Rate of Return is the Yield to Maturity (YTM). Ideally, to annualize this return, we use the equation earlier discussed under Annualizing Yields. However in the bond market, the convention is to *annualize by multiplying by a straight factor that converts the period to a year* i.e., a factor of 2 for semi-annual bonds. When this is done, the annualized yield obtained is called the Bond Equivalent Yield. Please note that the Bond Equivalent Yield always understates the true or effective yield on a bond. The YTM calculation takes into account the time value of money by including the timing of the cash flows and the related capital gain or loss that the investor will realize by holding the bond to maturity. The relationship among the coupon rate, current yield and YTM is as follows:

Market Price of Bond Expected relationship

(a) Par Coupon rate = Current yield = Yield to Maturity

(b) Discount Coupon rate < Current yield < Yield to Maturity

(c) Premium Coupon rate > Current yield > Yield to Maturity

The YTM formula assumes that the bond is held to maturity and all interim cash flows were reinvested at the YTM rate because it discounts all cash flows at that rate. The impact of the reinvestment assumption on the actual return of a bond varies directly with the bond's coupon and term to maturity. A higher coupon rate and term to maturity will increase the loss in value from failure to reinvest at the YTM. Therefore, a higher coupon or longer maturity makes the reinvestment assumption more important.

4. YIELD TO CALL (YTC)

For a bond that may be called prior to its stated maturity date, another yield measure is commonly quoted – ***the Yield to Call (YTC).*** The method of calculating YTC is the same as for calculating the YTM except that the cash flows used in the computation are those that are expected to occur before the first call date. YTC is, therefore, the yield that will make the present value of the cash flows up to first call date equal to the price of the bond i.e., assuming the bond is held to its first call date.

$$P = \frac{C}{1+y} + \frac{C}{(1+y)1} + \frac{C}{(1+y)2} + \ldots\ldots + \frac{C}{(1+y)n} + \frac{M*}{(1+y)n*}$$

M* = call price (in rupees)
n* = number of periods until first call date

Investors normally calculate the YTC and YTM and select the lower of the two as a measure of return. Again, like the YTM measure the YTC measure also assumes the following:

NOTE: (a) Reinvestment of interim cash flows at the YTC rate
(b) The bond is held to its first call date

5. HORIZON YIELD

This measures the expected rate of return of a bond that you expect to sell prior to its maturity. It is, therefore, a total return measure which, allows the portfolio manager to project the performance of a bond on the basis of a planned investment horizon, his expectations concerning reinvestment rates and future market yields. This allows the portfolio manager to evaluate which of several potential bonds considered for investment will perform best over the planned investment horizon. Using total return to assess performance over some investment horizon is called *Horizon Analysis* while the return calculated over the horizon is called *Horizon Yield or Return.*

The disadvantage of this approach for calculating return is that it requires the portfolio manager to make some assumptions about reinvestment rates, future yields and to think in terms of a specified period or horizon. It however enables the manager to evaluate the performance of a bond under different interest rate scenarios thereby assessing the sensitivity of the bond to interest rate changes.

5.5 BOND PRICE VOLATILITY

Interest rate changes do not affect all bonds equally. The longer the term to maturity of a bond, the greater the risk that the market price of the bond will fluctuate from the maturity value. In return for taking on the additional risk associated with longer-term bonds, investors expect to receive compensation in the form of increased yield. As a result, there is a direct link between maturity and yield, a link explained by the **yield curve**.

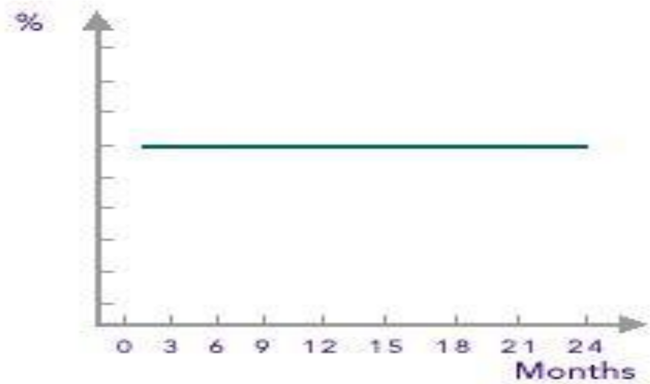
A yield curve is a graph in which interest rates are plotted against term to maturity for bonds of the same credit quality. Analysts, traders and investors study the shape of the yield curve carefully because it contains built-in expectations about future trends in interest rates.

Shape of the yield curve

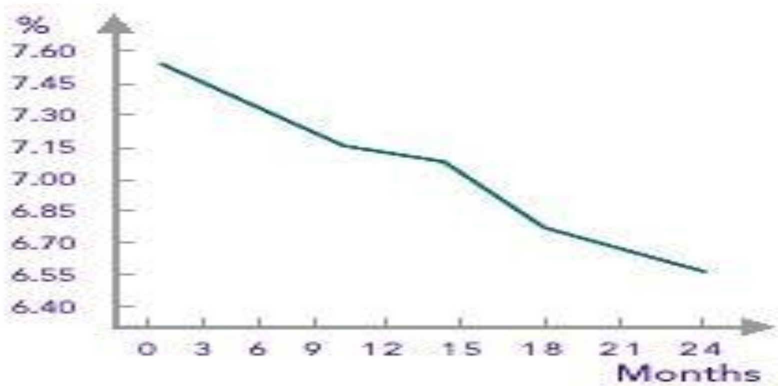
If you want to use the yield curve as an investment decision tool, you need to know that yield curves can take different shapes. More importantly, you need to know what each shape represents.

A normal (or upward-sloping) yield curve indicates that interest rates rise as maturities lengthen, i.e., short-term rates are lower than long-term rates. This is the normal type of interest rate environment, as opposed to the structure implied by a flat (horizontal) or downward-sloping (inverted) yield curve.

If the yield curve is steep, it means that by purchasing bonds with longer maturities, you can attain significantly increased bond yields (and income) compared to purchasing bonds with shorter maturities.



If, however, the yield curve is flat, the difference between short-term and long-term interest rates is relatively small. In such a situation, investors may prefer to remain in the short end of the yield curve and purchase bonds with shorter maturities.



An inverted or downward-sloping yield curve implies that the longer the bond's maturity is, the lower the available return will be. It indicates that investors expect interest rates to fall. An inverted yield curve is sometimes considered a sign of an imminent depression. Note that the relationship is not linear, the shape of the price-yield relationship is referred to as convex.

Bond price volatility is measured in terms of percentage change in price. Bond price volatility is influenced by more than yield behaviour alone. Factors that affect it are:
· Par value
· Coupon rate
· Term to maturity
· Prevailing interest rate

The following relationship exists between yield and bond price behavior:

- Prices move inversely to yield
- For a given change in yield, longer maturity bonds post larger price changes thus bond price volatility is directly related to term to maturity.
- Price volatility increases at a diminishing rate as term to maturity increases

- Price movements resulting from equal absolute increases or decreases in yield are not symmetrical. A decrease in yield raises bond prices by more than an increase in yield of the same amount lowers prices.
- Higher coupon issues show smaller percentage fluctuations for a given change in yield i.e., price volatility is inversely related to coupon.

5.6 MEASURES OF BOND PRICE VOLATILITY

A measure of interest rate sensitivity of a bond is called DURATION. There are three key measures and they are:
1. Macaulay duration
2. Modified duration
3. Effective duration

5.6.1 MACAULAY DURATION

Macaulay Duration *is a measure of the time flow from a bond*. It can also be likened to the weighted average number of years over which a security's total cash flows occur. The weightings used are the market value of the cash flows.

$$V = \sum_{i=1}^{n} PV_i$$

where:

- i indexes the cash flows,
- PV_i is the present value of the ith cash payment from an asset,
- t_i is the time in years until the ith payment will be received,
- V is the present value of all future cash payments from the asset.

Characteristics

1. Macaulay duration of a bond with coupon payment will be less than its term to maturity because of the interim cash flows.
2. There is an inverse relationship between coupon and duration i.e., the higher the coupon the lower the duration.
3. There is a positive relationship between term to maturity and Macaulay duration but duration increases at a decreasing rate with maturity.
4. There is an inverse relationship between YTM and duration, all other things being equal.
5. Sinking funds and call provisions have dramatic effect on a bond's duration

Illustration 5.9: Consider two bonds with the following features:

Face value	**₹1,000**	**₹1,000**
Maturity	**10 years**	**10 years**
Coupon rate	**4%**	**8%**

Calculate the Macaulay duration for each of the bonds assuming an 8% market yield.

Solution:

Bond A

Year	Cash Flow	PV @ 8%	PV of flow	PV as % of Price	PV as % of Price time weighted
1	40	0.9259	37.04	0.0506	0.0506
2	40	0.8573	34.29	0.0469	0.0938
3	40	0.7938	31.75	0.0434	0.1302
4	40	0.7350	29.40	0.0402	0.1608
5	40	0.6806	27.22	0.0372	0.1860
6	40	0.6302	25.21	0.0345	0.2070
7	40	0.5835	23.34	0.0319	0.2233
8	40	0.5403	21.61	0.0295	0.2360
9	40	0.5002	20.01	0.0274	0.2466
10	1040	0.4632	481.73	0.6585	6.5850
Total			731.58	1.0000	8.1193

Duration = 8.12 years

BOND B

Year	Cash Flow	PV @ 8%	PV of flow	PV as % of Price	PV as % of Price time weighted
1	80	0.9259	74.07	0.0741	0.0741
2	80	0.8573	68.59	0.0686	0.1372
3	80	0.7938	63.50	0.0635	0.1906
4	80	0.7350	58.80	0.0588	0.1906
5	80	0.6806	54.44	0.0544	0.2720
6	80	0.6302	50.42	0.0504	0.3024
7	80	0.5835	46.68	0.0467	0.3269
8	80	0.5403	43.22	0.0432	0.3456
9	80	0.5002	40.02	0.0400	0.3600
10	1080	0.4632	500.26	0.5003	5.0030
Total			1000.00	1.0000	7.2470

Duration = 7.25 years

It therefore seeks to measure the time characteristics of the bond.

5.6.2 MODIFIED DURATION

It measures the price volatility of a non-callable bond.

$$\textbf{Modified Duration} = \frac{\text{Macaulay Duration}}{1+\text{YTM}/n}$$

where :- YTM is the yield to maturity of the bond
n is number of payments per year

Please note that the greater the modified duration the greater the price volatility of the bond for small changes in yields. Specifically, an estimate of the percentage change in price equals the change in yield times the modified duration.

∴ dP_1 = - modified duration
dY P

Illustration 5.10: Consider a bond with Macaulay duration of 8 years, yield of 10%. Assume you expect the YTM to decline by 75 basis point (say from 10% to 9.25%). The modified duration is as follows:

Macaulay Duration = 8 years , YTM = $\frac{10}{100}$ = 0.1

Modified Duration = $\frac{\text{Macaulay Duration}}{1+\text{YTM}/n}$

Modified Duration = $\frac{8}{1+0.1/2}$ = **7.62 years**

The estimated percentage price change in the price of the bond is %age change in P = -7.62 X $\left(-\frac{75}{100}\right)$

= **5.72**

Note: The modified duration is always a negative for non-callable bonds because of the inverse relationship between price and yield. Also, modified duration provides a good estimate of price change for only small changes in yield of option free securities.
If you expect a decline in interest rates, you should increase the average modified duration of your bond portfolio to experience maximum price volatility. *Duration changes in a non-linear fashion with yield changes – a concept called **convexity***. It therefore, requires the recalculation and rebalancing as rate changes.

5.6.3 **EFFECTIVE DURATION** *is a direct measure of the interest rate sensitivity of a bond or any asset where it is possible to observe the market prices surrounding a change in interests.*

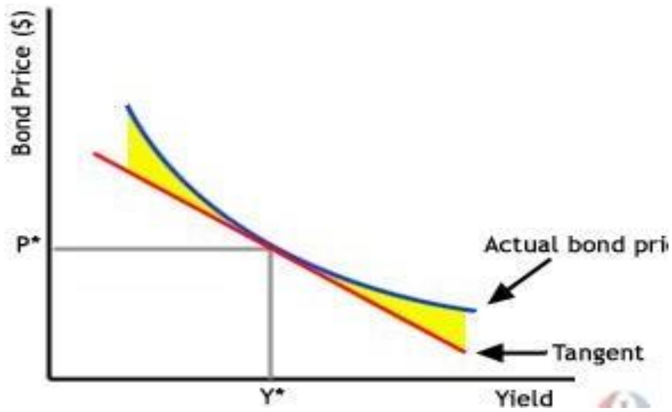
$$\text{Effective duration} = \frac{V_{-\Delta y} - V_{+\Delta y}}{2(V_0)\Delta y}$$

Where, Δ *y* is the amount that yield changes, and $V_{-\Delta y}$ and $V_{+\Delta y}$ are the values that the bond will take if the yield falls by *y* or rises by *y*, respectively. However this value will vary depending on the value used for Δ *y*.

Note:
1. It is possible to have an Effective duration greater than maturity as in the case of CMO.
2. It is also possible to compute a negative effective duration as in the case of bonds with embedded options .e.g. mortgage-backed securities.

5.7 CONVEXITY

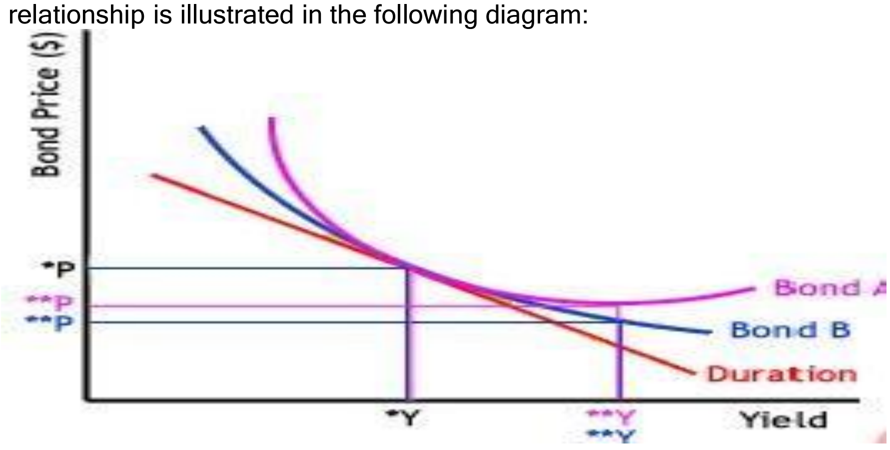
For any given bond, a graph of the relationship between price and yield is convex. This means that the graph forms a curve rather than a straight-line (linear). The degree to which the graph is curved shows how much a bond's yield changes in response to a change in price.



Furthermore, as yield moves further from Y*, the yellow space between the actual bond price and the prices estimated by duration (tangent line) increases. The convexity calculation, ∴ accounts for the inaccuracies of the linear duration line. This calculation that plots the curved line uses a Taylor series, a very complicated calculus theory that we won't be describing here. The main thing for you to remember about convexity is that it shows how much a bond's yield changes in response to changes in price.

PROPERTIES OF CONVEXITY

Convexity is also useful for comparing bonds. If two bonds offer the same duration and yield but one exhibits greater convexity, changes in interest rates will affect each bond differently. *A bond with greater convexity is less affected by interest rates than a bond with less convexity*. Also, bonds with greater convexity will have a higher price than bonds with a lower convexity, regardless of whether interest rates rise or fall. This relationship is illustrated in the following diagram:

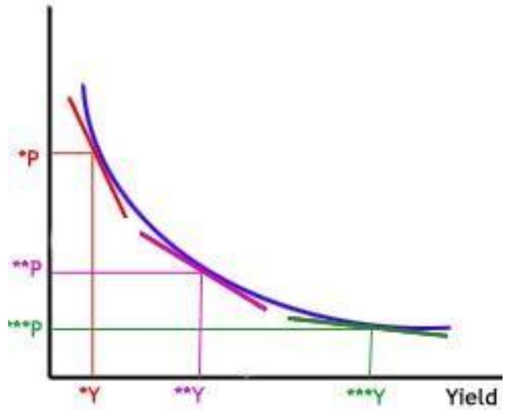


As you can see Bond A has greater convexity than Bond B, but they both have the same price and convexity when price equals *P and yield equals *Y. If interest rates change from this point by a very small amount, then both bonds would have approximately the same price, regardless of the convexity. When yield increases by a large amount, however, the prices of both Bond A and Bond B decrease, but Bond B's price decreases more than Bond A's. Notice how at **Y the price of Bond A remains higher, demonstrating that investors will have to pay more money (accept a lower yield to maturity) for a bond with greater convexity.

What Factors Affect Convexity?

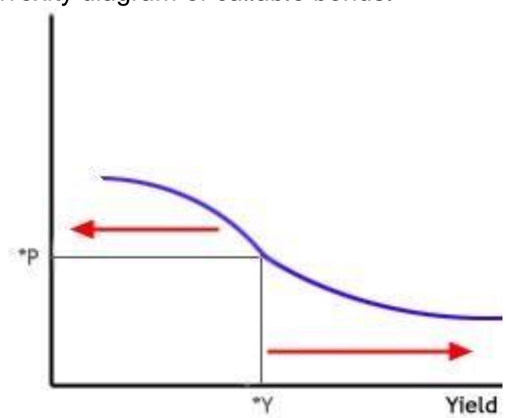
Here is a summary of the different kinds of convexities produced by different types of bonds:

1) The graph of the price-yield relationship for a plain vanilla bond exhibits positive convexity. The price-yield curve will increase as yield decreases, and vice versa. ∴ as market yields decrease, the duration increases (and vice versa).



2) In general, the higher the coupon rate, the lower the convexity of a bond. Zero coupon bonds have the highest convexity.

3) Callable bonds will exhibit negative convexity at certain price-yield combinations. Negative convexity means that as market yields decrease, duration decreases as well. See the chart below for an example of a convexity diagram of callable bonds.



Remember that for callable bonds, modified duration can be used for an accurate estimate of bond price when there is no chance that the bond will be called. In the chart above, the callable bond will behave like an option-free bond at any point to the right of *Y. This portion of the graph has positive convexity because, at yields greater than *Y, a company would not call its bond issue: doing so would mean the company would have to reissue new bonds at a higher interest rate. Remember that as bond yields increase, bond prices are decreasing and thus interest rates are increasing. A bond issuer would find it most optimal, or cost-effective, to call the bond when prevailing interest rates have declined below the callable bond's interest (coupon) rate. For decreases in yields below *Y, the graph has negative convexity, as there is a higher risk that the bond issuer will call the bond. As such, at yields below *Y, the price of a callable bond won't rise as much as the price of a plain vanilla bond. Convexity is the final major concept you need to know for gaining insight into the more technical aspects of the bond market. Understanding even the most basic characteristics of convexity allows you to better comprehend the way in which duration is best measured and how changes in interest rates affect the prices of both plain vanilla and callable bonds.

Bond Yield-to-Maturity

Imagine you are interested in buying a bond, at a market price that's different from the bond's par value. There are three numbers commonly used to measure the annual rate of return you are getting on your investment: (or loss)

Coupon rate: Annual payout as a percentage of the bond's par value.
Current Yield: Annual payout as a percentage of the current market price.
Yield-to-Maturity: Composite rate of return of all payouts, coupon and capital gain
(The capital gain or loss is the difference between par value and the price you actually pay.)

The yield-to-maturity is the best measure of the return rate, since it includes all aspects of your investment. To calculate it, we need to satisfy the same condition as with all composite payouts:

Whatever r is, if you use it to calculate the present values of all payouts and then add up these present values, the sum will equal your initial investment.

In an equation,

$$c(1 + r)^{-1} + c(1+r)^{-2} + \dots\dots + c(1 + r)^{-n} + B(1 + r)^{-n} = P$$

where,

c = annual coupon payment

n = number of years to maturity

B = par value

P = purchase price

For Example: The bond is selling for ₹950, and has a coupon rate of 7%; it matures in 4 years, and the par value is ₹1,000. What is the YTM?

The coupon payment is ₹70 (that's 7% of ₹1,000), so the equation to satisfy is

$$70(1 + r)^{-1} + 70(1 + r)^{-2} + 70(1 + r)^{-3} + 70(1 + r)^{-4} + 1000(1 + r)^{-4} = 950$$

YTM is greater than the current yield, which in turn is greater than the coupon rate. (Current yield is ₹$\frac{70}{950}$ = 7.37%). This will always be true for a bond selling at a discount. In fact, you will always have this:

Bond Selling At	Satisfies This Condition
Discount	Coupon Rate < Current Yield < YTM
Premium	Coupon Rate > Current Yield > YTM
Par Value	Coupon Rate = Current Yield = YTM

Illustration 5.11 Assume a yield to maturity of 8 percent. Compute the duration for the following bonds. Assume ₹100 par values. For the 12% coupon bond, compute the duration using the two duration formulas. Which formula is easiest to compute?

(a) 10 years, zero coupon

(b) 10 years, 8 percent coupon

(c) 10 years, 12 percent coupon

YTM = 8% ; par = ₹100 ; DUR = ?

a. $N = 10$; $C = 0$; $DUR = 10$

Duration equals the maturity of **10.**

b. $N = 10$; $C = 8\%$

$$DUR_{par\ bond} = \frac{(1+y)[1-(1+y)^{-n}]}{y}$$

$$DUR = \frac{(1.08)[1-(1.08)^{-10}]}{0.08} = 7.2469$$

c. $N = 10$; $C = 12\%$

$$P = c\left[\frac{(1-(1+y)^{-n})}{y}\right] + \left[\frac{Par}{(1+y)^{n}}\right]$$

$$P = 12\left[\frac{(1-(1.08)^{-10}}{0.08}\right] + \left[\frac{100}{(1.08)^{10}}\right] = 126.84$$

1) $$DUR = \frac{\left[\frac{c(1+y)}{y}\right]\left[\frac{1-(1+y)^{-n}}{y}\right] + n\left[\frac{par - c/y}{(1+y)^{n}}\right]}{P}$$

$$\mathrm{DUR} = \frac{\left[\frac{12(1.08)}{0.08}\right]\left[\frac{1-(1.08)^{-10}}{0.08}\right] + 10\left[\frac{100-12/0.08}{(1.08)^{10}}\right]}{126.84}$$

$$\mathrm{DUR} = 6.7442$$

4) $$\mathrm{DUR}_{\text{par bond}} = \frac{(1.08)[1-(1.08)^{-10}]}{0.08} = 7.2469$$

$$\mathrm{DUR}_{\text{non par}} = n - [n - \mathrm{DUR}_{\text{par}}]\left[\frac{c/P}{y}\right]$$

$$\mathrm{DUR}_{\text{non par}} = 10 - [10 - 7.24689]\left[\frac{12/126.84}{0.08}\right]$$

$$\mathrm{DUR}_{\text{non par}} = 6.7442$$

Illustration 5.12 In problem 5.11, assume that yields change from 8 to 9 percent. Work out the exact change in price and compare it with the change in price predicted by duration. Explain the difference. Assume Rs 100 par values.

a. $$\frac{(P_1 - P_0)}{P_0} = \frac{(42.24 - 46.32)}{46.32} = -0.0880$$

- (Change in yield)(DUR) = (-0.01)(10) = **-0.10**

The duration approximation is slightly more than 1 percent too large in absolute value.

b. $$\frac{(P_1 - P_0)}{P_0} = \frac{(93.58 - 100)}{100} = -0.0642$$

- (Change in yield)(DUR) = (-0.01)(7.2468) = **-0.0725**

The duration approximation is 0.83 percent too large in absolute value.

c. $$\frac{(P_1 - P_0)}{P_0} = \frac{(119.25 - 126.84)}{126.84} = -0.0598$$

- (Change in yield)(DUR) = (-0.01)(6.7442) = **-0.0674**

The duration approximation is 0.76 percent too large in absolute value.

Illustration 5.13: Compute the duration of a portfolio composed of a ten-year, zero coupon bond and a ten-year, 8 percent coupon bond. Yield to maturity is 8%. For simplicity assume each bond has a par value of $100. Suppose that equal dollar amounts are invested in the two bonds.

Solution:

Bond 1:

N = 10; *C* = 0; *FV* = 100; *I/YR* = 8; *PV* = ?

PV = 46.319

$DUR_{\text{zero coupon bond}}$ = n = 10

Bond 2:

N = 10; *C* = 8; FV = 100; *I/YR* = 8; *PV* = ?

PV = 100

$$\text{DUR}_{\text{portfolio}} = \frac{(P_1)(\text{DUR}) + (P_2)(\text{DUR}_2)}{P_1 + P_2}$$

$$DUR_{portfolio} = \frac{(46.319)(10) + (100)(7.247)}{146.319} = 8.1185$$

Illustration 5.14 A perpetual bond has a coupon of $6 and a yield to maturity of 6 percent. Work out the actual percentage change in price and the duration approximation in the following three cases:

(a) The yield decreases by 1 percent

(b) The yield increases by 1 percent

(c) The yield increases by 8 percent.

$$P_0 = \frac{6}{0.06} = Rs100.00$$

$$\text{DUR}_{\text{perpetual}} = \frac{1+y}{y} = \frac{1.06}{0.06} = 17.67$$

a. $$P_1 = \frac{6}{0.05} = Rs120.00$$

% change = $\frac{120 - 100}{100} = 0.2 = 20\%$

$$\%\Delta P \approx (Duration)\Delta y = 17.67 \times 0.01 = 17.67\%$$

b. $P_1 = \dfrac{6}{0.07} = Rs85.71$

% change = $\dfrac{85.71 - 100}{100}$ = - 0.1429 = -14.29%

$$\%\Delta P \approx (Duration)\Delta y = 17.67 \times (-0.01) = -17.67\%$$

c. $P_1 = \dfrac{6}{0.14} = Rs42.86$

% change = $\dfrac{42.86 - 100}{100}$ = - 0.5714 = - 57.14%

$$\%\Delta P \approx (Duration)\Delta y = 17.67 \times (-0.08) = -141.36\%$$

Compute the duration for the following bond. Coupon = Rs 9, Par = Rs 100, maturity = 12 years. The yield to maturity is 10%.

$$DUR = N - \left[N - DUR_{par}\right]\left[\frac{c/P}{y}\right]$$

P = 93.1863

$$\begin{aligned} DUR &= 12 - [12 - 7.495061]\left[\frac{9/93.1863}{0.10}\right] \\ &= 12 - [4.504939]\left[\frac{9/93.1863}{0.10}\right] \\ &= 12 - [4.504939][0.965807] \\ &= 12 - 4.350012 \\ &= 7.64938. \end{aligned}$$

Student Activity

1. You purchase 225 shares of Anchor Corporation Common stock at ₹44.80 per share. A few months later, you sell the shares at ₹53.20. Your stock broker charges 2% commission on round lots and 3% on odd lots. Calculate the total cost, the proceeds and the gain or loss on the transaction.
2. A four year bond with the 7% coupon rate and maturity value of ₹1,000 is currently selling at ₹905. What is its yield to maturity?
3. Calculate the duration for bond A and bond B with 7 per cent and 8 per cent coupons having maturity period of 4 years. The face value is ₹1,000. Both the bonds are currently yielding 6 per cent.

Summary

- Stocks and bonds are financial instruments for investors to obtain a return and for companies to raise capital. Stocks offer an ownership stake in the company and bonds are akin to loans made to the company.

- Stocks of a company are offered at the time of an IPO (Initial Public Offering) or later equity sales. The company offers investors an ownership stake by selling stocks. Stocks can be either common stock or preferred stock. Preferred stock is further divided into participating and non-participating preferred stock.

- Bonds are loans offered at a fixed interest rate. When a company believes that it can raise capital cheaper by borrowing money from banks, institutional investors or individuals, they may choose to offer interest-paying corporate bonds. With bonds, an investor is promised a fixed return.

Glossary

Bonds: In finance, a bond is an instrument of indebtedness of the bond issuer to the holders. It is a debt security, under which the issuer owes the holders a debt and, depending on the terms of the bond, is obliged to pay them interest (the coupon) and/or to repay the principal at a later date, termed the maturity date. Interest is usually payable at fixed intervals (semiannual, annual, sometimes monthly).

Stock: The stock (also capital stock) of a corporation constitutes the equity stake of its owners. It represents the residual assets of the company that would be due to stockholders after discharge of all senior claims such as secured and unsecured debt. Stockholders' equity cannot be withdrawn from the company in a way that is intended to be detrimental to the company's creditors

YTM: The rate of return anticipated on a bond if held until the end of its lifetime. YTM is considered a long-term bond yield expressed as an annual rate. The YTM calculation takes into account the bond's current market price, par value, coupon interest rate and time to maturity.

Answers to Self Assessment Questions

1. TC = ₹10,292.80, Proceeds = ₹11,717.30, Gain = ₹1,424.50
2. YTM = 9.8%
3. A = 3.631 years, B = 2.592 years

Review Questions

1. Mr. Ajay purchases 650 shares of Gulf Corporation Common stock at ₹44.25 per share. A few months later, he sells the shares at ₹57.29. Your stock broker charges 3% commission on round lots and 1.5% on odd lots. Calculate the total cost, the proceeds and the gain or loss on the transaction.

2. Determine the total cost and proceeds of each purchase and sale. Include regular commission of 0.20 per share and odd lot differential of 0.125 per share.
   - Purchased 300 shares of Company A at 106.10
   - Purchased 550 shares of Company B at 5.20
   - Purchased 200 shares of Company C at 30.07

3. A bond with the face value of ₹1,000 pays a coupon rate of 9 per cent. The maturity period is 9 years (a) Find out the approximate yield to maturity (b) current yield and the nominal interest rate.

4. Consider a bond selling at a par value of ₹1,000 with 7 years to maturity and 8 per cent coupon payment. Calculate (a) the bonds duration and
(b) If the yield to maturity increases to 9%. What would be the price change?

5. Determine Macaulay's duration of a bond that has a face value of₹1,000 with 10 per cent annual coupon rate and 3 years term to maturity. The bond's yield to maturity is 12 per cent.

6. Ann's bond portfolio manager advises her to buy a 7 year, ₹5,000 face value bond that gives 8% annual coupon payments. The appropriate discount rate is 9%. The bond is currently selling at ₹4,700. Should Ann adhere to the manager's advice?

Further Readings

Sanchethi and Kapoor, *Business Mathematics*

R.S. Bhardwaj, *Business Mathematics*, Excel Books.

Zvi Bodie, *Security Analysis and Portfolio Managements*

UNIT 6 FOREIGN EXCHANGE

Unit structure

- Introduction
- Foreign exchange market
- Rate of exchange
- Summary
- Keywords
- Review Questions
- Further Readings

Learning Objectives

After reading this unit you should be able to understand:

- Foreign exchange market
- Direct and indirect rates of exchange
- Conversion and chain rule

6.1 INTRODUCTION

Foreign exchange refers to a foreign currency or claims relating to a foreign currency. In India, a 10 British pound currency note, or a 10 travelers' cheque or a demand draft drawn on a London bank would be termed as foreign exchange. The term foreign exchange, broadly speaking, includes bank deposits denominated in a foreign currency, foreign currency itself (bills and coins) and other short term claims on foreigners expressed in foreign currency. Most foreign exchange transactions, however, involve purchases and sales of bank deposits denominated in foreign currencies.

6.2 FOREIGN EXCHANGE AND FOREIGN CURRENCY

For an Indian, *foreign currency means any currency other than Indian currency. As per the **Foreign Exchange Regulations Act 1999***, of India, foreign exchange includes all deposits, credits and balances payable in any foreign currency, and any travelers' cheques, drafts, letters of credit and bills of exchange, expressed or drawn in Indian currency but payable in any foreign currency; and also any instrument payable at the option of the drawee or holder thereof or any other party thereto, either in Indian currency or in foreign currency or partly in one and partly in the other.

Foreign exchange may also refer to the mechanism by which the currency of one country gets converted into the currency of another country.

The foreign exchange market is a market where money denominated in one currency is traded with money denominated in another currency. The need for such trading arises due to buying and selling of goods, services of goods, services rendered to foreigners, investment in short term and long term securities across international boundaries, foreign bilateral and multilateral assistance, and so on.

The primary function of the foreign exchange market is to facilitate trade between different countries and to enable investment by one country in another. A knowledge of foreign exchange market operations and mechanism is necessary for any fundamental understanding of international financial management.

6.3 FOREIGN EXCHANGE MARKET

The foreign exchange market is not a physical place. It is a network of banks, foreign exchange brokers and dealers whose function is to bring buyers and sellers together. It is not confined to any one country but is dispersed throughout leading financial centres such as London, New York, Paris, Zurich, Amsterdam, Tokyo, Toronto, Milan and Frankfurt. *Trading is done by telephone/telex. Computer monitors display minute-to-minute information – current exchange rate ranges of all major currencies for delivery at various dates.* Every major bank round the world exhibits the exchange rate ranges at which it is probably willing to trade currencies with other banks. Business is conducted generally by oral communication – written confirmation occurs later.

Before making a final deal, the buyer or the seller bank tries to get directly from other banks (selling or buying) a farmer price bid. As huge sums are involved in foreign exchange dealings, even a small difference may mean a lot of money.

Because of the time difference between countries, and because of the nature of foreign currency transactions, viz. over the counter transactions, the foreign exchange market can be considered to be a twenty-four hour market. Business hours overlap around the world. As one centre closes, another centre will be still trading.

The volume of business varies substantially according to market conditions and according to the time of day. The market is the deepest or most liquid only in the European afternoon where the world's largest trading centres – London, New York, Frankfurt and Chicago – are all open together.

High trading volume is accompanied by narrow spreads. Dealers can reverse their position easily due to high liquidity; so their risk is small. Spreads on minor currencies are wider than on the major currencies.

6.4 PARTICIPANTS IN THE FOREIGN EXCHANGE MARKET

The participants in the foreign exchange market include *retail customers, commercial banks, foreign exchange brokers and the central banks – in the case of India the Reserve Bank of India*. **Retail customers**, which include a king list of exporters, importers, hedgers, speculators, arbitrageurs, borrowers, investors, travelers, multinational corporations, public sector units, embassies and others, transact with commercial banks. **The commercial banks** have been authorized by the Reserve Bank of India to deal in foreign exchange. They are referred to as authorized dealers/authorized persons. Authorized dealers buy and sell foreign exchange for their customers. Traders entering the foreign exchange in order to exchange currencies seldom transact with each other directly. Each trader deals with a bank usually in his own country. For this purpose, banks hold foreign exchange inventories in the form of deposits with banks in foreign countries. While the authorized dealers can transact business directly with each other, they take the help of foreign exchange brokers for interchanging their currencies – to be precise for interchanging their deposits. **The foreign exchange brokers** bring together banks that sell foreign exchange with those who buy foreign exchange. **The central bank**, in order to bring about stability in the exchange rate, or to maintain a target rate, also intervenes in the market occasionally.

When the foreign currency is in short supply, the domestic currency is likely to depreciate in value in relation to foreign currency. In order to prevent depreciation of the domestic currency, the central bank intervenes in the foreign exchange market by making available the scarce foreign currency. This action on the part of the central bank is likely to prevent further fall in the value of the domestic currency.

Conversely, if the value of the domestic currency appreciates as a result of abundant supply of foreign currency, the central bank can absorb the surplus from the market by buying the foreign currency at a specified rate. This action is likely to restore the exchange rate to its previous level. However, unless the intervention takes place at the right time and in substantial amount the policy may not be successful and this has got to be supplemented by other monetary policies.

6.5 SOURCES OF SUPPLY AND DEMAND FOR FOREIGN EXCHANGE

The most important sources of supply of foreign exchange is export of goods and services. In countries like *Hong Kong* and *Singapore* re-exports also play a very important role in earning foreign exchange. The banking insurance and transport services also earn foreign exchange through the services rendered by them to their foreign customers. Foreign direct and portfolio investments and foreign deposits by the residents earn foreign exchange in terms of dividends, profits, interest income, etc. Another important source through which foreign exchange is earned is the tourist expenditure. Foreign tourists on their vacation spend lots of money in other countries. This adds to the foreign exchange earnings of the country. Persons who are employed outside their countries usually send foreign currencies to their relatives in their home countries in the form of private remittances. These remittances increase the foreign exchange kitty. Foreign diplomatic missions spend lots of money in those countries where they are located. This increases the foreign exchange earnings of the countries where they are located. A major source of foreign exchange inflow comes from foreign aid or foreign assistance in the form of loans, grants, etc. Repayment of loans and interest payments by foreigners is another source which increases the foreign exchange inflow. Countries sometimes approach multilateral financial institution like the International Monetary Fund (IMF) and the World Bank and other regional development banks for financial assistance. Grant of loans by these institutions increases the supply of foreign exchange.

Importers of goods and services demand huge amounts of foreign exchange. Similarly, payments are made for the services rendered by the foreign banking, insurance and transport sectors. If the residents of a country want to invest abroad either in the equity or portfolio investment, their act increases the demand for foreign exchange. The need to pay profits dividends, interest, etc. to foreign investors increases the demand for foreign exchange. If residents want to go abroad for a pleasure trip, education, medical treatment or business or want to send money to their relatives abroad, the demand for foreign exchange increases.

Demand for foreign exchange is a *derived demand*. Foreign exchange is not demanded for its own sake. It is demanded in the process of buying or selling something else – a product or service.

The foreign exchange market provides a useful service to hedgers by allowing hedgers of all nationalities to get rid of net asset or net liability position in currencies they do not want to own or owe. Who helps them? There are speculators who are willing to take a net position in a foreign currency – long (buying more than needed) i.e. having more or short (selling more than what is in possession), i.e., having a net liability position.

The existence of a foreign exchange market does not guarantee that speculation would be profitable. It only makes speculation possible for those willing to take that chance.

6.6 RATE OF EXCHANGE

Each nation's money has a price in terms of other nation's money. *Exchange rate is the price of one currency in terms of another*. Knowledge of exchange rates is important because they connect the price systems of different countries. Suppose a firm is interested in purchasing equipment which is available in the USA for $10,000 and in England for £6,200 on March 26, 1999. At the then prevailing exchange rates, the firm would have to spend ₹4,21,900 obtaining it from the USA and 4,22,034 for obtaining it from the U.K. Thus the firm is better off by obtaining it from the USA.

Bankers are in a position to give customers the rates of any currency desired by them. Theoretically for ***n*** currencies, the number of possible exchange rates would be ***n(n -1)***. There are roughly more than *180* currencies in the world. The exchange rates of important currencies are published by financial newspaper like the Wall Street Journal, New York, The Financial Times, London, and International Financial Statistics published by the IMF. Indian financial newspaper like the Economic times, The Financial Express, Business Standard also publish exchange rates for a limited number of currencies relevant to Indian businessmen.

An exchange rate, *a ratio between the currencies of two countries*, may be stated in two ways. They are known as *exchange rate quotations*.

(1) Rupee price of foreign currency or number of units of foreign currency per unit/s of local currency. For example, ₹100 = $2.3702 or 1.4691. This method in which the unit of home currency is kept constant and the exchange rate is expressed as so many units of foreign currency is called *'indirect quotation'*.

(2) Price of the rupee in terms of foreign currency or number of units of local currency per unit of foreign currency. For example, $ 1 = Rs 42.19 or £1 = Rs 60.07. This method under which the exchange rate is expressed as the price per unit of foreign currency in terms of the home currency is known as '*direct quotation'*. Under direct quotation, the number of units of foreign currency is kept constant and any change in exchange rate is expressed by changing the value in terms of rupees.

It is important to make certain that the quotation is clearly understood when dealing with exchange rates. From August 2, 1993, India switched over to direct method of quotation. It is easier for the public to understand what the cost of foreign exchange they need would be and what would the rupee receipts of foreign exchange earned by them is.

6.7 SYSTEMS OF EXCHANGE RATES

There are various systems of calculating and expressing the rate of exchange, such as the Fixed Rate Floating Rate, Flexible Rate, etc. These are briefly described below:

6.7.1 Fixed rate of Exchange

a) Mint Par: Countries like the U.K., the U.S.A. etc., while on gold standard, a monetary system which was abandoned in 1931, based the value of their respective currencies on a specified quality of fine gold, making their currencies freely convertible into gold at the rate so fixed. This fixed relationship of the currencies with a common denominator, that is gold, led to fixed parities between them, known as the mint par of exchange. It was held that under this arrangement the exchange rate between two currencies would vary, if at all, only slightly from the mint par of exchange, since any movement away from the mint par would make it profitable to convert the currency into gold, export the gold to a foreign centre and then convert it into a foreign currency.

b) Specie Points: Under gold standard, gold could be exported or imported freely. But the export or import of gold involved a certain amount of expenditure in shipping and insuring it, and the expenditure so incurred set a margin on either side of the mint par of exchange, called the specie points, to the extent of which the rate of exchange might vary before there was any movement of gold. When, however, the exported gold was sold in a foreign currency, there naturally, was a larger demand for the concerned foreign currency raising its value above the mint par, while when the sale process were sold there were larger offers of that currency, reducing its value back towards the mint par.

In a reverse process, gold would be imported. But the import of gold would necessitate the purchase of foreign currency with which to obtain the gold, and this would cause the value of that currency to appreciate back towards the mint par.

c) Advantages/Disadvantages of Fixed Exchange Rates: The great advantages of a fixed exchange rates system is that fixed rates of exchange foster the growth of international trade, for traders are then able to calculate accurately the amounts they will have to pay or will receive in their own currency for the goods imported or exported by them. Another advantage of fixed exchange rates is that the competitive depreciation of exchange rates, which tend to occur when floating exchange rates are in operation, may be avoided.

The great disadvantage of a fixed exchange rates system is that lack of confidence in the currency would cause heavy drains upon the country's reserves of gold and foreign currencies, and a shortage of reserves would make the country take restrictive measures, such as import control or monetary control, to put the balance of payments right and maintain the rate of exchange. Another disadvantage of the system is that the currencies may be overvalued or undervalued, throwing the pattern of exchange rates out of alignment and thereby necessitating major adjustments in them.

6.7.2 Floating/Flexible Rates of Exchange

a) **Floating Rates**: Under a floating exchange rates system there are no fixed parities, and the rates of exchange are allowed to float, i.e., fluctuate freely without official intervention. It has been argued in favour of floating rates that the fixing of parities with gold or otherwise is, after all, arbitrary, and that under a fixed exchange rates system the currency may be overvalued or undervalued, leading to a persistent adverse of favourable balance of payments, whereas by allowing the rate to float in accordance with the demand for and supply of the currency, it may be fixed by market operations at the level of the true international value of the currency, leading to growth in international trade.

A floating exchange rates system implies a complete suspension of parities or any attempt at pegging the rates, i.e., artificially confining through official intervention the movement of the rates of exchange within certain predetermined limits. In practice, however, the necessity of occasional official intervention to correct too erratic fluctuations in the rate cannot be altogether ruled out.

b) **Flexible Rate**: Under a flexible exchange rates system, the rate of exchange is allowed to fluctuate in response to the market conditions of demand and supply.

Floating and flexible exchange rates are more or less alike. However, a distinction is attempted by some. According to them, a floating exchange rate implies a complete absence of official intervention or of any attempt at pegging the rate, whereas a flexible exchange rate may be subject to relatively frequent changes in the exchange parities or may be subject to the pegging of the rate.

6.7.3 Intermediary Arrangements

Between the fixed and the freely flexible rates of exchange, there may be at least four intermediary arrangements – the flexible exchange rate with government intervention; the wide band; the adjustable peg; and the sliding or crawling peg.

a) **Flexible Rate with Government Intervention:** The flexible exchange rate with government intervention presupposes that the government can recognize transitory influences which should not be allowed to move the exchange rate or set in motion factors which would lead to resource reallocation. The advocates of government should spend reserves or draw on the IMF during a deficit period, and restock or repay during a surplus one. They further suggest that the government should allow the rate to move in response to other influences such as changes in demand and supply. But the flexible exchange rate advocates oppose intervention on the ground that the governments are no better at distinguishing temporary from far-reaching effects than private speculators.

b) **The Wide Band:** The wide band proposal was put forward by Keynes, the famous British economist. The proposal envisaged freely fluctuating exchange rates within a band wide enough sto have effects on resource allocation, but not so wide as to discourage, because of risk, international economic intercourse. In other words the movements of the exchange rates within the band should not be so wide as to discourage trade and investment, and not too limited to promote adjustment. The wide and proposal lies between fixity and flexibility of the exchange, and contemplates plus or minus 10 per cent of parity instead of $2\frac{1}{4}\%$ permissible under the IMF Agreement.

c) **The Adjustable Peg:** The adjustable peg is a fixed rate of exchange which is changed from time to time. This position was virtually adopted by the IMF Agreement at Bretton Woods in 1944, when it was agreed that small movements in the exchange rate were permissible at any time and larger movements, in the event of fundamental disequilibrium. The adjustable peg is said to stimulate destabilising speculation by providing speculators a one-way option. When a currency is in trouble, there is no chance of its going up but a considerable chance of its going down. It may then be safe to speculate against it. Moreover, small changes made successively in the rate of exchange may lead to bigger changes and thus a national crisis of adjustments may ensue, calling for international arrangements to avert the crisis, as in the case of the Basel Agreement in 1961.

d) **The Crawling Peg:** The crawling or sliding peg is a more subtle scheme which has been devised to contain fluctuations in the rates of exchange. Under this scheme, the exchange rates are not pegged at one point, but are free to move, though to a limited extent, in each period. The implication is that successive periods would result in a discrete change, but the change in any one period is not large enough to encourage speculation. The crawling / sliding peg system has not been taken up by business or government circles.

6.7.4 Other Exchange Rates Systems

a) Unitary Exchange Rates System: When there is only one official rate of exchange in a country for all types of transactions, a unitary exchange rates system is deemed to be in operation. This system is preferable to a differential exchange rates system for a currency, for the latter system tends to undermine confidence in the currency. A unitary exchange rate system is one of the objectives of the IMF, and many leading members of the Fund, including India, have adopted this system.

b) Multiple Exchange Rates System: This is an arrangement under which different rates are permitted for a country's currency for different types of transactions. The IMF is opposed to multiple currency practices.

c) Two-Tier Exchange Rates: A country having a system of multiple exchange rates may have a controlled rate of exchange for its currency for a certain transaction and a free market rate for other transactions. Or it may have one rate for exports and a lower rate for imports, or one rate for mercantile transactions and a lower rate for transactions and a lower rate for transactions of a capital nature to prevent any outflow of capital. When this happens the rates are known as two-tier exchange rates.

6.7.5 Other Exchange Rate

Equilibrium Rates of Exchange: When the value of a currency in terms of another currency reflects its purchasing power parity and the country maintains an equilibrium in its balance of payments, the rate of exchange between the two currencies is said to be the equilibrium rate of exchange.

6.8 FACTORS INFLUENCING EXCHANGE RATES

Since the establishment of the IMF, the international monetary system seemed to be evolving towards fixed rates till coming into force in April 1978 of its Second Amendment, Nevertheless, the exchange rate between any two currencies may, and in fact does, vary from day to day within the limits set under the IMF Agreement; that is up to 2 $\frac{1}{4}$% on either side of the parity. The rate of exchange, i.e., the market price of a currency, is determined, as in all other cases, by the demand for and the supply of the currency. The factors that tend to influence such demand and supply are indicated in the following paragraphs.

a) **Commercial Transactions:** If imports exceed exports, the demand for foreign currencies rises, forcing up the values thereof in terms of the home currency, that is to say the value of the home currency is depreciated in terms of the concerned foreign currencies. If exports exceed imports, there is a greater demand for the home currency, forcing its price up in terms of the concerned foreign currencies. This may also be called balance of payments factor influencing the rate of exchange.

b) **Investments:** When the government or an industrial concern of a country undertakes a project in another country, the investment in that country in connection with the project necessitates the purchase of the currency of that country, thereby weakening the values of the home currency in terms of that currency. Conversely, investments in this country will cause an increased demand for the home currency and push up its value.

c) **Government Loans and Grants:** When a country grants loans or makes gifts to another country, the currency of the lender country so provided may be used to buy other currencies in the foreign exchange market for purchases to be made from those countries. Such purchases tend to weaken the value of the lender country's currency. When the loan is repaid or when interest on the loan is paid, such payment tends to strengthen the currency of the lender country.
Tied Loans: Efforts to counteract this adverse effect on the lender country's currency have given rise to what is known as tied loans, that is, loans with stipulation that these must be used to buy goods produced by the lender country.

d) **Transactions with International Institutions:** when a country subscribes its contribution to the IMF or the International Bank for Reconstruction and Development(IBRD) or to any of its associates, and that currency is lent out to another member country, such lending tends to weaken the exchange rate of the subscriber country's currency.

e) **Arbitrage:** Arbitrage in foreign exchange consists in simultaneous buying and selling of one or more currencies in different exchange markets so as to make a profit out of the differences, in the exchange rates of the currencies at different centres. Arbitrage operations may be classified as under.

- *Two-Point Arbitrage*: when the arbitrageur finds a spread in the price of the currency of his own country in two markets, generally his own and one abroad, that is, when the transaction is confined to two markets, it is a single or direct or two-point arbitrage. For example, if the rates of exchange are-
  ₹1 = $ 2
  ₹1 = 500 francs
  $1= 125 francs
  And the dollar-franc rates moves to $1 = 100 francs, then the arbitrageur may buy 1000 francs in London and sell them in New York, making a profit of $1.

- *Three-Point arbitrage or Cross Arbitrage*: When the buying and selling involves three markets-for instance, purchasing francs against dollars in New York, then selling the francs for pounds in London and then selling the pounds against dollars in New York-we have what is known as a three-point arbitrage.

- *Arbitrage in Space* refers to transactions in foreign exchange to take advantage of discrepancies between rates quoted at the same moment at different markets.

- *Arbitrage in Time* refers to transaction to take advantage of discrepancies between forward margins for different maturities.

- *Interest Arbitrage* refers to the movement of funds from low interest to a high-interest centre, provided that a profit is likely even after allowing for the cost of forward cover, and that the centre to which funds are moved is regarded as politically and economically stable.

- *Arbitrage transactions* have the effect of leveling down discrepancies in the rates of exchange obtaining in different centres, thereby making two or more centres, though physically separate, a single market where only one price exists for the same commodity. This is sometimes referred to as Interest Differential factor influencing the rate of exchange.

f) **Short-Term Capital Movements**: Short-term capital is attracted to a centre with a comparatively high rates of interest, causing an increased demand for the currency of that country and hence a rise in its rate of exchange. On the other hand, capital flows out of a centre where the interest rate is comparatively low, resulting in a fall in the rate of exchange of the currency of that country.

g) **Confidence in the Currency**: High rate of interest, no doubt, attracts foreign capital. But there would be no movement of capital into a country unless there is, generally speaking, confidence in that country's currency. This factor is also known as the *Inflation Differential factor* influencing the rate of exchange.

- *Leads and Lags*: If it is feared that the currency of a country may be devalued, then the importers in the country, who have payments to make in foreign currency, would endeavour to obtain that currency quicker than usual. In similar circumstances, the exporters in the country, who have claims upon foreign importers, would defer selling foreign currencies until after the devaluation. This hastening to obtain foreign currency, as in the former case, is known in the *exchange market as leads* and the deferment of selling foreign currencies is known as *lags*. Both leads and lags weaken the rate of exchange.

h) **Technical Factors**: The technical factors influencing the rate of exchange include keeping open positions at weekends, window-dressing operations at the year-end, cover operations for the returns that banks have to submit to the authorities, etc.

6.9 COMMERCIAL RATES OF EXCHANGE

The rates of exchange quoted in the foreign market are described below.

6.9.1 Spot Rate

The normal rate quoted in the foreign exchange market is the spot rate. This rate is quoted for transactions where the foreign currency bought or sold is to be received or delivered immediately. It is the basic rate from which all other rates of exchange are calculated.

6.9.2 TT Rates

This is spot rate used for remittances from one country to another by telegraphic transfers, i.e. for transactions in which the receiving and paying over of the amounts involved are made almost simultaneously and no question of interest is involved.

Valuer Recompense: This *French* phrase means in English *value compensated*. In some foreign exchange transactions, such as remittances by TT or draft, the value is received at one centre and paid over at another centre almost on the same day, involving no loss of interest. Such transactions are based on the principle valuer recompenses, that is, value compensated.

Value Date: The term value date is a specific date on which the foreign exchange bought or sold, has to be received or delivered and its price in the local currency, is to be paid. It is the date on which a payment of fund or an entry in an account becomes effective and/or subject to interest.

Value-dated Transactions: Transactions in foreign exchange which specify the date on which the foreign currency bought and sold has to be delivered and the price thereof in the local currency is to be paid are value-dated transactions.

In the event of non-delivery of the foreign currency or of non-payment of the value in the local currency on the specified date, the defaulting party becomes liable to pay interest for the period from the value-date to the date of payment. The value date is usually the same at both centres, so that the delivery of the foreign currency and the payment in the local currency take place on the same day. Hence, the phrases value here and there, value compensated, valuer recompense.

6.9.3 Currency Rate/Currency Quotation

When the rate of exchange is quoted a fixed number of units of the home currency in terms of a varying number of units of a foreign currency, it is called the currency or certain or indirect rate of exchange.

Currency Quotation: an exchange rate quotation expressed as a fixed number of home currency units in terms of a varying number of foreign currency units is called o foreign currency or indirect quotation. e,g.,

₹100 = £ 1.2437

Or ₹100 = $ 2.450

Here, the amount in rupees(home currency)Price quotation is fixed, while the amount in pound sterling and dollar (foreign currency) varies.

In India the usual practice is to quote the rate of exchange in the currency/certain/indirect method in terms of ₹00.

6.9.4 Pence Rate/Direct Quotation

A rate of exchange quoted as a fixed number of a foreign currency units in terms of a varying number of home currency units is direct or pence rate of exchange, and a quotation so expressed is a home currency or direct quotation. For instance, if the rupee-sterling rate is quoted at £ = 7₹.35, i.e. in terms of so many (i.e. a varying number of) rupees to a pound, it is a pence or direct rate. As present the State Bank of India follows this method in quoting its rate of exchange.

This rate of exchange derived the name of pence rate from the former practice followed in London of quoting the exchange rate of the pound in terms of the currencies based on silver. As the price of bar silver used to be quoted in London in pence per ounce, it was more convenient, for the purpose of calculation, to quote the exchange rate in pence. This practice is no longer followed, but the name continues to be in use.

6.9.5 Buying and Selling Rates

a) A foreign exchange transaction is either **a purchase or a sale transaction**, i.e., a transaction which involves either buying or selling of a foreign currency. For the buying and selling of foreign currencies, there are different rates. The price in terms of the home currency at which a banker is willing to buy a foreign currency is the buying rate, and the price, also in terms of the home currency, at which the banker will sell a foreign currency is the selling rate, between the two currencies.

b) Two-Way Quotations: The buying and selling rates of a currency in terms of another currency vary, and the two are quoted together as under:

₹100 = £ 1.2437 – 1.1821

&

£ 1 = $1.64 – 1.6316

In the above quotations the rates

₹100 = £ 1.2437 and £ 1 = $ 1.64 are the buying rates, and the rates

₹100 = £ 1.1827 and £1 = $ 1.63 are the selling rates.

A quotation with a pair of rates, i.e., the buying and selling rates, is known as a two-way price or quotation.

c) Dealing Spread: The buying and selling rates of exchange are, as a rule, different, and the difference between the two is referred to as the dealing spread, or simply the spread.

d) Middle Rate: The middle rate of a currency lies exactly half way between its market buying and selling rates. For instance, if New York is quoted at £ = $ 1.64 – 1.63, the middle rate would be $1.635 per 1£

The middle rate should not, however, be referred to as the mean or average rate. The rate is calculated by first finding the spread, then dividing it by 2, and finally by adding the result to the market selling rate or subtracting it from the buying rate.

6.9.6 Buying Rates

There are four types of buying rates:

a) **TT Buying Rate:** This is the rate at which a banker buyer buys a TT or draft issued on him by an overseas branch or correspondent. It is the rate at which the foreign currency, in which the TT or draft is drawn, is converted into the home currency before making payment thereof. Here, the receipt of the value in foreign currency and its payment in the home currency take place, theoretically at least, on the same day; but, in practice, on the second working day. Hence, no interest factor is considered to be involved in such transactions.

   There are two kinds of TTs in foreign exchange transactions, viz., TT (clean), where the instruction by cable is simply to pay a certain sum of money to a certain person, and TT(documentary), where the instruction by cable is to pay a certain sum of money to a certain person on production of certain specified documents. Accordingly, there are two TT buying rates:

   i. *TT (clean) rate* – applicable to TTs (clean) in which the handling of documents is not involved; and
   ii. *TT (doc.) rate* – applicable to TTs (doc.) in which the handling of documents is involved.

   The TT (clean) rate is the basic rate from which all the other buying rates are worked out. The TT (doc.) rate is higher than the TT (clean) rate and is, therefore, less favourable for customers.

b) **OD (on Demand) Buying Rate:** It is the TT (clean) buying rate loaded with interest for the transit period. This rate applies to transactions, such as purchase or negotiation of export bills, in which there time-lag between the giving out of value at one end being compensated therefore, at the other end, thereby entitling the purchasing/negotiating bank to interest for the transit period.
   Normal Transit period is the period not exceeding one month from the date of purchase or negotiation of an export bill till the date of first presentation, the negotiation banker becomes eligible for overdue interest, i.e. interest for the overdue period from the date of first presentation till the date of payment.

c) **Long Buying rate:** This is the rate used for discounting usance or long bills, and is calculated by loading the TT (clean) buying rate with interest for the transit period plus the period of usance plus the days of grace, where applicable.
   If the long rate is based on the OD buying rate, interest for the usance period and the days of grace, where applicable, is to be added.

d) **Tel Quel Rate or T.Q.Rate:** The words tel quel means such as it is. The is a long buying rate with a difference, and is quoted for discounting usance bills having only a broken period of unsance to run. The long buying rate is calculated by loading the OD buying rate with interest for the full usance period, while a tel quell rate is calculated by loading the OD buying rate with interest only for the broken period, plus in either case, interest for the grace period, if any. For instance, if a unsance bill drawn on 25-3-14 for 90 days is presented for discounting on 25-4-14, then the tel quell rate would be the ruling OD rate plus interest for the broken period of 59 days plus days of grace, if any…

6.9.7 Selling Rates

Sale transactions in foreign exchange are classified into two groups, viz., clean sales i.e., issue of TTs, MTs, drafts circular letters of credit, traveler's cheque, etc., and sales arising out of import bills. Accordingly, there are two types of selling rates;

i. TT Selling Rate, the rate applicable to clean sales, and

ii. BC (Bill collection) Selling Rate, the rate applicable to payment of import bills.

6.10 EXCHANGE RATES MAXIMS

a) Buy High, Sell Low: This maxim applies to foreign currency / indirect quotations. Here the word *high means the higher rate of exchange and the word low means the lower rate of exchange*; and the maxim implies that it is advantageous for the banker to buy at the higher rate and sell at the lower rate available. For example, if a banker purchases an export bill for £10,000 @ Rs 100 = £1.2437, he pays to the exporter about Rs 8,04,052 but if he had purchased the bill @ Rs 100 = £1.1821, he would have to pay about Rs 8,04,595 about, i.e. Rs 54.3 more. Conversely, if the banker sells £10000 @ Rs 100 = 1.2437 instead of the lower rate of Rs 100 = 1.1821, he receives Rs 8,04,595 less from the buyer.

This maxim is sometimes expressed as Take more, Give less.

b) Buy Low sell High: This maxim is applicable to direct/home currency quotations. It implies that it is profitable for a banker to purchase foreign currency at the lower rate and sell it at the higher. For instance, if a banker buys an export bill for £10,000 @ £1 =₹57.70 instead of the lower rate, say, £1 = ₹55.70, he pays to the exporter ₹5,77,000 instead of ₹5,57,000, i.e., ₹20,000 more. On the other hand, if he buys an import bill for £1 = 55.70, he receives ₹20,000 less.

c) The Better the Bill, the Lower the Rate: The earlier a bill of exchange is payable the better is it considered to be. Thus, a DP bill, i.e. a bill payable on presentation, is better than a usance bill, i.e., a bill payable after a specified period; or a short bill, i.e. a usance bill having only a few days to run to maturity irrespective of the original tenor of it, is better than a long bill, i.e. a bill having a usance of more than a month. As indicated above, an OD buying rate, i.e., the rate of demand bills, is lower than a long rate, i.e. the rate for usance bills, or a tel quell rate is lower than a long rate. Thus, a customer gets a better or worse rate according as the bill is payable on demand or after a short or a long period.

6.11 CROSS-CURRENCY RATES

Not all the currencies are traded daily. Even if two specific currencies are not being traded in a particular market, bankers are in a position to give customers the rates of any currency desired by them. This rate can be obtained by the method known as the chain rule, and the rate is referred to as the cross rate. Let us take the middle rate and assume no transaction cost.

Illustration: Let us assume that the following rates are known:

If INR 48.9150	= 1USD	(say Indian Rupees per USD)
And if 1 USD	= 0.66842 GBP	(say GBP per USD)
1 GBP	= ?INR	(say Indian Rupees per GBP)

The first equation ends with the currency USD and the same currency is started in the second equation. Similarly, the second equation ends with GBP and third one starts with the same currency, thus forming a chain. Multiply the items on the LHS (left hand side) and the RHS (right hand side) separately. The LHS = 48.9150 and the RHS = 0.66842. Divide the LHS by RHS. The answer is ₹ 73.1800.

The same answer can be obtained in another way. Let us do the simple general mathematical calculations. Represent the first line as INR/USD, the second line as GBP/USD, third line as INR/GBP.

Divide the first line by the second:

INR/USD = INR

Using the information given

48.9150/0.66842 = INR 73.1800

We can also multiply the first two lines if we use the inverse of the second line.

Similar exercises can be done for any number of currencies. Such information can be displayed in a matrix form as given Table.

Table: Cross-currency Rates

US $(USD)	-	1.0364	0.6685	1.5240	1.2245	1.5283	3.6727	3.7515
Euro(EUR)	0.9648	-	6450	1.4704	1.1815	1.4745	3.5435	3.6196
GB pound Sterling	1.4959	1.5504	-	2.2798	1.8318	2.2862	5.4940	5.6120
Swiss Franc (CHF)	0.6562	0.6801	0.4386	-	0.8035	1.0028	2.4099	2.4617
Yen(JPY, Per 100 yen)	0.8166	0.8464	0.5459	1.2445	-	1.2480	2.9992	3.0637
Canadian $ (CAD)	0.6543	0.6782	0.4374	0.9972	0.8013	-	2.4032	2.4548
UAE Dirham(AED)	0.2723	0.2822	0.1820	0.4150	0.3334	0.4161	-	0.0047
Saudi Rail (SAR)	0.2666	0.2763	0.1782	0.4062	0.3264	0.4074	0.9790	-

Let us work out an example from the information in given table

1.0364 EUR = 1 USD

1 USD = 0.6685 GBP

1 GBP = ? EUR

In other words, 0.6685 GBP = 1.0364 EUR

Hence 1 GBP = 1.0364/0.6685 = 1.5504 EUR.

6.12 DIRECT CALCULATION

If the equivalent amounts of two currencies or the respective gold contents per unit, i.e., parties, of two currencies are given, the rate of exchange between the two currencies may be found by the simple method of the Rule of Three.

Illustration 6.1 If a bank in India paid ₹3,00,000 to a customer and claimed reimbursement by airmail by £19,798.50 from the remitting bank, what was the rate of exchange in terms of ₹1,000?

Solution:

This is a clean inward remittance. Hence, the rate of exchange should be a TT (Clean) buying rate. The calculation of the rate would thus be along the following lines:

If for ₹3,00,000 the bank claimed £19,798.50, then how many pounds would it claim for ₹100?

The rate would be – $\frac{100 \; X \; 1979.50}{300000}$ = £ 65,995

or ₹100 = **£ 65,995**

Illustration 6.2 If the gold contents (parities) or U.S. dollar and Indian rupee were –

$ 1 = 0.888675 gramme of fine gold, and

₹1 = 0.118489 gramme of fine gold

What would be the par value (rate of exchange) of the U.S. dollar in terms of ₹100?

Solution:

The working out of the required par value will be on the following lines:

If the gold contents of a rupee were 0.118489 gramme, what would be the total gold contents of ₹100? And then how many dollars would have equivalent gold contents on the basis of 0.888675 gramme per dollar?

$0.118489 \times \frac{100}{0.888675} = 13.33$

The required parity or rate of exchange between Indian rupees and U.S. dollars would be: ₹100 = **$13.33.**

6.13 CONVERSELY

If the gold contents of a currency unit, the price of gold and the parity of the currency in terms of another currency are known, then the gold contents of one of the currencies under a revised parity may be determined as under:

Illustration 6.3 The gold contents of Indian rupee were declared as 0.118489 gramme of fine gold when the official price of gold was U.S., $35 per ounce and the rupee-dollar parity was U.S. $1 =₹7.50. With the recent increase in the official price of gold to U.S. $42.22 per ounce, if the Government of India were to re-fix the rupee-dollar parity at U.S. $1 = ₹7.20, what would be the revised gold contents of the rupee?

Also, find out the percentage (correct to the second decimal) of the revaluation/devaluation of the rupee represented by the change in its gold contents.

Solution:

The working out of the problem would be along the following lines:

Gold contents of Indian rupee when the official price of gold were U.S. $ 35 per ounce	- 0.118489 gm
With the change in the official price of gold, contents of the rupee would be –	
$\frac{35 \times .118489\ gramme}{42.22}$	- 0.098263 gm
With the refixing of the rupee-dollar parity @ $ 1 = ₹ 7.20, the gold contents of the rupee would be –	
$\frac{7.50 \times 0.982263\ gramme}{7.20}$	- 0.102319 gm
Thus, the gold contents of the rupee have fallen from 0.118489 gm to 0.103219 gm, i.e. by	- 0.016170 gm
Hence, there has been a devaluation of the rupee at percentage	- 13.6488
$.016170 \times \frac{100}{.118489}$ or 13.65 (correct to the second decimal)	

6.14 CALCULATION OF RUPEE/FOREIGN CURRENCY EQUIVALENT

Given the buying and selling rates of rupees in terms of another currency, the rupee equivalent of a specified amount of that currency or the foreign currency equivalent of a specified sum of rupees may be found by the method of the Rule of Three. Where interest for transit period and discount are charged, the banks concerned at both ends realize their commission, and/or if the bill is rediscounted at the prevailing discount rate at the foreign centre, all these are to be added to find the equivalent sum.

Illustration 6.4 If a bank in India quotes its rates for U.S. dollars as under:

	Selling Rates		Buying Rates	
	TT	BC	TT	OD
₹ 100 =	£ 12.65	£ 12.62	£ 12.75	£ 12.80

What amount in rupees would the bank recover from its customer to remit $ 25,000 to New York by airmail transfer?

Solution :

This is a clean sale for the bank. Hence, the rate applicable is the TT selling rate, viz, ₹100 = $ 12.65.

∴ the rupee equivalent of $ 25,000 would be :

$$\frac{25000 \times 100}{12.65} = ₹\mathbf{1,97,628.45}$$

Illustration 6.5 If a bank in India opens a documentary letter of credit in favour of an exporter in London in pound sterling equivalent to ₹50,000, what would the amount in sterling be for the proposed L/C, if the bank's rates for sterling are :

	Selling Rates		Buying Rates	
	TT	BC	TT	OD
₹ 100 =	£ 6.5475	£ 6.5375	£ 6.6055	£ 6.6520

Solution:

Here, the transaction involves the handling of documents. Hence, the rate applicable is the BC selling rate, viz., ₹100 = £ 6.5375.

∴ the amount of the proposed L/C in sterling equivalent to ₹50,000 would be :

$$\frac{25,000 \times 6.5375}{7.20} = \mathbf{£\ 3,925.50}$$

Illustration 6.6 An exporter requests you to negotiate a draft for £ 20,000 drawn on London at 60 days sight. Assuming that –

Clean TT rates on London are 5.5280 – 5.5605

You rediscount the draft in London at 7 ½ % up to 90 days

Your discount charges are ½ % per annum

Your commission is ¼ %

London bank's charges are $\frac{1}{8}$ %

Transit period is 10 days

Interest for transit period is 7%

What rupee amount will you pay to the exporter on negotiation of the drafts?

Solution:

Here, the transaction is the buying of an export bill drawn at 60 days sight. Hence, the rate the bank would use in purchasing the bill would be the long buying rate, i.e. the TT buying rate loaded with interest for the transit period, re-discount charges, bank's own discount charges and the bank's as well as the London bank's commission. The principle would be: Buy high, sell low. The calculation would be as under, taking 360 days to a year:

TT (clean) buying rate on London – ₹ 100 = £ 5.5605

Add London discount charge @ 7 ½ % + bank's own discount charge @ ½ %

i.e. $\frac{5.5605 \times (7\frac{1}{2} + \frac{1}{2} \times 60)}{100 \times 360}$ = £ 0.0741

Add transit interest @ 7% for 10 days

$\frac{5.5605 \times 7 \times 10}{100 \times 360}$ = £ 0.0108

Add Bank's commission @ ¼ % = £ 0.0139

Add London Bank's com. @ $\frac{1}{8}$ % = £ 0.0069

Total = £ 5.6662

or rounded off to a multiple of 5 => £ 5.6660

The rupee equivalent of the export bill for £ 20,000 would be:

$\frac{100 \times 20,000}{5.6660}$ = **₹3,52,982.70**

Illustration 6.7: On 26th June, a bank had purchased an export bill for £ 95,000 at £ 5.3730 for ₹100. On 3rd July, when the bill was presented to the overseas buyer, it was returned unpaid. A telex message to this effect was received by the bank on 6th July. On contacting the exporter the bank was authorised to debit the bill amount to his account value 16th July.

In the meantime, on 2nd July the Indian Rupee was revalued, and the following new rates for merchants were fixed:

Selling Rates		Buying Rates	
TT	BC	TT	OD
5.3510	5.3410	5.3955	5.4305

Interest on overdue bill at 18% per annum

What will be the profit/loss to the exporter on this transaction?

Solution:

The required profit or loss to the exporter on the transaction will depend on whether the amount debited to his account in adjustment is less or more than the amount received by him on the purchase of the bill by the bank. The working out of the profit/loss will be as under:

Step I. The amount received in rupees by the exporter on the bank's purchasing the bill –

$$\frac{95{,}000 \times 100}{5.3730} = ₹17{,}68{,}099.76$$

Step II. The reconversion into rupees of the bill amount in adjustment of the returned bill being a ready sale transaction, the TT selling rate on the revaluation of the rupee, viz, $5.3510 for ₹100, is to be used. The rupee equivalent will thus be :

$$\frac{95{,}000 \times 100}{5.3510} = ₹17{,}75{,}369.09$$

Add: interest @ 18% p.a. for 20 days (from 26th) June to 16th July, one day being excluded on the amount under step I, viz., ₹17,68,099.76.

$$\frac{17{,}68{,}099.76 \times 18 \times 20}{100 \times 365} = ₹\ 17{,}438.79$$

Total amount to be debited to the exporter's account = ₹ 17,92,807.88

This amount being in excess of the amount received by the exporter at the time of purchase of the bill by ₹ 24,708.12, there will be **loss of ₹ 24,708.12** to the exporter on the transaction.

6.15 CALCULATION OF RUPEE-OTHER CURRENCY RATES

The calculation of cross rates between the Indian rupee and any currency other than pound sterling is based on the previous day's quotation of the rate between sterling and that currency in the London market. As there is a time-lag of at least one day in the working out of the rate, banks in India usually, but not necessarily, add to the buying rate, or deduct from the selling rate, a margin by way of cover for any likely fluctuation in the London rate in the meantime.

Illustration 6.8 If a bank in Kolkata acquires U.S. dollars in the London market when the rates of exchange quoted are –

₹ 100 = £ 6.5574 (selling rate)

£ 1 = $ 1.7203 (selling rate),

What will be the rates of exchange between the rupee and the dollar?

Solution:

As pound sterling will have to be sold to acquire U.S. dollars the calculation is to be based on the selling rates, and the required rate is to be worked out by the Chain Rule method.

? US $ = Rs. 100

If ₹ 100 = £ 6.5547

And if £ 1 = U.S. $ 1.7203

$$\frac{1.7203 \times 6.5574 \times 100}{100} \text{ or } 11.2806$$

The required rate is ₹100 = **$11.28**

Note I: To get the TT buying rate, the exchange profit is to be added to this rate.

Note II: To get the TT selling rate, the exchange profit, brokerage, etc., are to be deducted from the rate.

Note III: To get the BC selling rate a further deduction of the exchange difference is to be made from the rate.

Illustration 6.9: Canadian dollars are quoted in London at 2.6169 – 2.6215 spot

Assuming that –

(a) You have to charge 50 cents by way of cushion on the London rate.
(b) You can sell sterling to the Reserve Bank at £ 5.2851 per ₹100.
(c) You need an exchange profit of ¼ %
(d) The transit period is 12 days
(e) Expenses on stamps, commission, etc., amount to ¼ %, and
(f) The interest on bills is 6%

At what rate would you buy a sight bill on Canada expressed in Canadian dollars? Explain each step in detail to show the basis on which the rate has been arrived at.

Solution:

Here, two quotations are given:

Quotation I: ₹100 = £ 5.2851

i.e., the rate at which the bank can sell pound sterling to the Reserve Bank by way of cover for the purchase of the bill.

Quotation II: London buying rate for Canadian dollars, viz., £ 1 = Can. $ 2.6215

To this rate is to be added the cushion, viz. 0.5000

Final sterling Can. Dollar rate £ 1 = Can. $ 3.1215

From these two quotations, the rupee-Canadian dollar cross rate is to be found by the Chain Rule method as under:

? Can. $ = Rs 100

If £ 1 = Can. $ 3.1215

and If Rs 100 = £ 5.2851

$$\frac{100 \ X \ 5.2851 \ X \ 3.1215}{100 \ X \ 1} = 16.4974$$

Thus, the rupee-Can. Dollar TT buying rate will be ₹100 = Can. $ 16.4974

To this rate are to be added the given loading factors

Add exchange profit @ ¼ %	Can. $ 0.0412
Add interest @ 6% for 12 days (16.4974 X 12 X $\frac{}{100}$ x $\frac{}{360}$)	Can. $ 0.0330
Add cost of stamp, com, etc. @ ¼ %	Can. $ 0.0412
The rate for buying the bill will be i.e., ₹100 = Can. $ 16.61	Can. $ 16.6128

Illustration 6.10: Assuming that the sterling-dollar parity is £ 1 = U.S. $ 2.60 and the rupee-sterling parity is £ 1 = ₹18.95, what would be the rupee-dollar parity if Britain decides to devalue the pound sterling in terms of the dollar by 24% and India simultaneously decides to revalue the rupee in terms of pound sterling by 9 per cent?

Express your answer in terms of the rupee equivalent of one U.S. dollar correct to the fourth decimal place.

Solution:

We have to find out the sterling-dollar after the devaluation and the rupee-sterling parity after the revaluation, and then, on the basis of the new parities, the rupee-dollar parity.

STEP I: The sterling-dollar parity (rate) before devaluation is:

£ 1 = U.S. $ 2.60

After devaluation by 24%, the value of £ 1 becomes $\frac{76\ (i.e., 100-24)}{100}$ of the former £ 1

Hence, the new sterling-dollar parity will be:

£ 1 = \$2.60 x $\frac{76}{100}$ or \$ = 1.976 ...(1)

STEP II: The rupee-sterling parity before revaluation is: £ 1 = Rs 18.95

After revaluation by 9% the value of ₹1 becomes $\frac{109}{100}$ of the former ₹1.

Hence, the new rupee-sterling parity will be a fewer number of rupees per £ i.e.,

£ 1 = ₹18.95 x $\frac{100}{109}$ = ₹17.3852 ...(2)

STEP III: On the basis of the new rates (1) and (2), the cross rate between the rupee and the dollar by the Chain Rule method will be:

? Rupees = U.S. $1

If US $ 1.976 = £ 1

and If £ 1 = ₹17.3852

$$17.3852 \times \frac{1}{1.976} = 8.7982$$

The new rupee-dollar parity will be **$ 1 = ₹8.7982.**

Student Activity

1. If a banker in Mumbai wants to dispose of U.S. dollars in the London market when the rates are – ₹100 = £ 6.6007 (buying rate)
   and £ 1 = 1.7205 (buying rate). What would be rupee-dollar rate?
2. What best rate will you quote for a sight bill for Dutch guiders 75000 drawn on Amsterdam when the rates are as under:
   London on Amsterdam – 6.2625 – 6.2725 spot
   Mumbai on London – 5.3385 spot
   (Interest for the transit period may be taken at 10½% for the 15 days' transit and commission at ¼ % on the transaction).
3. Given £ 1 = U.S. $ 1.90 and £ 1 =₹15.20. What would be the rupee-dollar parity if pound is devalued 22% in terms of dollar and rupee is revalued in terms of pound by 8%? Express your answer in the rupee equivalent on one U.S. dollar correct to the fourth place of decimal.
4. You, as a foreign exchange dealer of your bank, are informed that your bank has sold a TT on Rome for Italian lire 50,000,000 at the rate of Italian lire 7540 =₹100. You are required to cover the transaction through London or New York, whichever course offers you a more profitable rate. The rate on the day are:
   Bombay – London 5.3075 – 5.3100
   Bombay – New York 12.9975 – 13.0025
   London – Rome 1424.50 – 1425.50
   New York – Rome 0.171950 – 0.172050 (expressed in U.S. cents to one Italian lira)
   Will you cover the transaction through London or New York? What will be the exchange profit on the transaction?

Summary

- The exchange of one currency for another, or the conversion of one currency into another currency. Foreign exchange also refers to the global market where currencies are traded virtually around-the-clock. The term foreign exchange is usually abbreviated as "forex" and occasionally as "FX."
- The foreign exchange market (forex, FX, or currency market) is a global decentralized market for the trading of currencies. The main participants in this market are the larger international banks. Financial centers around the world function as anchors of trading between a wide range of multiple types of buyers and sellers around the clock, with the exception of weekends. The foreign exchange market determines the relative values of different currencies.

Glossary

Foreign Exchange: Foreign exchange is the system by which commercial nations discharge their debts to each other.

Exchange Rates: an exchasnge rate (also known as a foreign-exchange rate, forex rate, FX rate or Agio) between two currencies is the rate at which one currency will be exchanged for another. It is also regarded as the value of one country's currency in terms of another currency.

Answers to Self Assessment Questions

1. ₹ 100 = US $ 11.35
2. ₹ 100 = DG 33.72
3. ₹ 9.68
4. 540.14

Review Questions

1. M/S ABC exporters have presented to you documents for USD 48,573.56 under an irrevocable letter of credit which provides for TT reimbursement of drawings thereunder. Assuming USD/INR is being quoted in the local inter-bank as 49.4300/4500 and one month forward is at par, what will be the exchange rate to be quoted to the customer and the rupee amount payable to him bearing in mind the following;
   Exchange margin of 0.10% is to be loaded
   Rate of interest: 10% p.a.
   Transit period: 5 days
   Out-of-pocket expenses of ₹400 to be recovered?

2. Bank A wants to sell in the local market U.S. $ 50,000,000 delivery cash. Calculate the rupee amount which the bank will receive from the buyer, assuming that the rates are quoted as under:

U.S. dollar/sterling	2.10345	2.10395
Sterling/rupee	5.3875	5.3900

   Brokerage @ 0.03%

3. You bought Italian lira 120 million TT from your customer at ₹100 – Italian lira 10320. You wish to maximize your profit by covering yourself either in pound sterling or in U.S. dollars, whichever is more profitable and dispose of the resultant pound sterling or U.S. dollars in the local exchange market.
   Assuming that the Italian lira spot were quoted in London as under:
   £ 1 – Italian lira 1575.00 U.S. $ 1 = Italian lira 862.50 and assuming that local lira-bank rates for sterling and U.S. dollars were as under:
   Spot Rs 100 - £ 65,050
   ₹100 – U.S. $ 1,18,600

In which currency would you cover the Italian lira in London and what will be your profit? Show full working.

4. A Bombay bank receives a sight bill drawn on an Indian importer for £ 5480 claused payable by TT on London without loss in exchange. Bombay on London TT is quoted at 55470.90. Show the rupee amount to be collected of the banker is to adjust his collection charges of ½ in the rate quoted to the importer.

5. Your customer requests you to issue a demand draft on Hamburg for Dmks 25,000. Assuming U.S. $ - Dmks is quoted in London market as –

Spot Rs 100 =	U.S. $ 1,03,800	U.S. $ 1,03,850
One month	U.S. $ 1,03950	U.S. $ 1,0,4000
Two months	U.S. $ 1,04,100	U.S. $ 1,04,150
Three months	U.S. $ 1,04,400	U.S. $ 1,04,450

What rate would you quote to your customer, bearing in mind that you are required to make an exchange profit of ¼ % on the transaction? Exchange profit to be included in the rate. Exchange rate to be nearest to the second decimal.

Further Readings

Levi, Maurice, *International Finance*, McGraw Hill

Buckley, A, *Multinational Finance*

Zvi Bodie, *Security Analysis and Portfolio Management*